INSTRUCTOR'S RESOURCE MANUAL

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SOUTHERN ILLINOIS UNIVERSITY - EDWARDSVILLE

CALCULUS

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CHAPTER

1 Preliminaries

1.1 Concepts Review

1. rational

2. $\sqrt{2};\pi$

3. real

4. theorems

Problem Set 1.1

1. $4-2(8-11)+6=4-2(-3)+6$
$=4+6+6=16$

2. $3[2-4(7-12)]=3[2-4(-5)]$
$=3[2+20]=3(22)=66$

3. $-4[5(-3+12-4)+2(13-7)]$
$=-4[5(5)+2(6)]=-4[25+12]$
$=-4(37)=-148$

4. $5[-1(7+12-16)+4]+2$
$=5[-1(3)+4]+2=5(-3+4)+2$
$=5(1)+2=5+2=7$

5. $\frac{5}{7}-\frac{1}{13}=\frac{65}{91}-\frac{7}{91}=\frac{58}{91}$

6. $\frac{3}{4-7}+\frac{3}{21}-\frac{1}{6}=\frac{3}{-3}+\frac{3}{21}-\frac{1}{6}$
$=-\frac{42}{42}+\frac{6}{42}-\frac{7}{42}=-\frac{43}{42}$

7. $\frac{1}{3}\left[\frac{1}{2}\left(\frac{1}{4}-\frac{1}{3}\right)+\frac{1}{6}\right]=\frac{1}{3}\left[\frac{1}{2}\left(\frac{3-4}{12}\right)+\frac{1}{6}\right]$
$=\frac{1}{3}\left[\frac{1}{2}\left(-\frac{1}{12}\right)+\frac{1}{6}\right]$
$=\frac{1}{3}\left[-\frac{1}{24}+\frac{4}{24}\right]$
$=\frac{1}{3}\left(\frac{3}{24}\right)=\frac{1}{24}$

8. $-\frac{1}{3}\left[\frac{2}{5}-\frac{1}{2}\left(\frac{1}{3}-\frac{1}{5}\right)\right]$
$=-\frac{1}{3\left[\frac{2}{5}-\frac{1}{2}\left(\frac{5}{15}-\frac{3}{15}\right)\right]}$
$=-\frac{1}{3}\left[\frac{2}{5}-\frac{1}{2}\left(\frac{2}{15}\right)\right]=-\frac{1}{3}\left[\frac{2}{5}-\frac{1}{15}\right]$
$=-\frac{1}{3}\left(\frac{6}{15}-\frac{1}{15}\right)=-\frac{1}{3}\left(\frac{5}{15}\right)=-\frac{1}{9}$

9. $\frac{14}{21}\left(\frac{2}{5-\frac{1}{3}}\right)^2=\frac{14}{21}\left(\frac{2}{\frac{14}{3}}\right)^2=\frac{14}{21}\left(\frac{6}{14}\right)^2$
$=\frac{14}{21}\left(\frac{3}{7}\right)^2=\frac{2}{3}\left(\frac{9}{49}\right)=\frac{6}{49}$

10. $\frac{\left(\frac{2}{7}-5\right)}{\left(1-\frac{1}{7}\right)}=\frac{\left(\frac{2}{7}-\frac{35}{7}\right)}{\left(\frac{7}{7}-\frac{1}{7}\right)}=\frac{\left(-\frac{33}{7}\right)}{\left(\frac{6}{7}\right)}=-\frac{33}{6}=-\frac{11}{2}$

11. $\frac{\frac{11}{7}-\frac{12}{21}}{\frac{11}{7}+\frac{12}{21}}=\frac{\frac{11}{7}-\frac{4}{7}}{\frac{11}{7}+\frac{4}{7}}=\frac{\frac{7}{7}}{\frac{15}{7}}=\frac{7}{15}$

12. $\frac{\frac{1}{2}-\frac{3}{4}+\frac{7}{8}}{\frac{1}{2}+\frac{3}{4}-\frac{7}{8}}=\frac{\frac{4}{8}-\frac{6}{8}+\frac{7}{8}}{\frac{4}{8}+\frac{6}{8}-\frac{7}{8}}=\frac{\frac{5}{8}}{\frac{3}{8}}=\frac{5}{3}$

13. $1-\frac{1}{1+\frac{1}{2}}=1-\frac{1}{\frac{3}{2}}=1-\frac{2}{3}=\frac{3}{3}-\frac{2}{3}=\frac{1}{3}$

14. $2+\frac{3}{1+\frac{5}{2}}=2+\frac{3}{\frac{2}{2}-\frac{5}{2}}=2+\frac{3}{\frac{7}{2}}$
$=2+\frac{6}{7}=\frac{14}{7}+\frac{6}{7}=\frac{20}{7}$

15. $\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)=\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2$
$=5-3=2$

16. $\left(\sqrt{5}-\sqrt{3}\right)^2=\left(\sqrt{5}\right)^2-2\left(\sqrt{5}\right)\left(\sqrt{3}\right)+\left(\sqrt{3}\right)^2$
$=5-2\sqrt{15}+3=8-2\sqrt{15}$

17. $3\sqrt{2}\left(\sqrt{2}-\sqrt{8}\right)=3\sqrt{4}-3\sqrt{16}$
$=3\cdot 2-3\cdot 4$
$=6-12=-6$

18. $2\sqrt[3]{4}\left[\sqrt[3]{2}+\sqrt[3]{16}\right]=2\sqrt[3]{8}+2\sqrt[3]{64}$
$=2\cdot 2+2\cdot 4$
$=4+8=12$

19. $\left(\frac{7}{4}+\frac{1}{2}\right)^{-2}=\left(\frac{7}{4}+\frac{2}{4}\right)^{-2}=\left(\frac{9}{4}\right)^{-2}$
$=\frac{1}{\left(\frac{9}{4}\right)^2}=\frac{1}{\frac{81}{16}}=\frac{16}{81}$

20. $\left(\frac{1}{\sqrt{2}}-\frac{5}{2\sqrt{2}}\right)^{-2}=\left(\frac{2}{2\sqrt{2}}-\frac{5}{2\sqrt{2}}\right)^{-2}$
$=\left(-\frac{3}{2\sqrt{2}}\right)^{-2}$
$=\frac{1}{\left(-\frac{3}{2\sqrt{2}}\right)^2}=\frac{1}{\frac{9}{8}}$
$=\frac{8}{9}$

21. $(3x-4)(x+1)=3x^2+3x-4x-4$
$=3x^2-x-4$

22. $(2x-3)^2=(2x-3)(2x-3)$
$=4x^2-6x-6x+9$
$=4x^2-12x+9$

23. $(3x-9)(2x+1)=6x^2+3x-18x-9$
$=6x^2-15x-9$

24. $(4x-11)(3x-7)=12x^2-28x-33x+77$
$=12x^2-61x+77$

25. $(3t^2-t+1)^2$
$=(3t^2-t+1)(3t^2-t+1)$
$=9t^4-3t^3+3t^2-3t^3+t^2-t+3t^2-t+1$
$=9t^4-6t^3+7t^2-2t+1$

26. $(2t+3)^3=(2t+3)(2t+3)(2t+3)$
$=(4t^2+12t+9)(2t+3)$
$=8t^3+12t^2+24t^2+36t+18t+27$
$=8t^3+36t^2+54t+27$

27. $\frac{x^2-4}{x-2}=\frac{(x-2)(x+2)}{x-2}=x+2$

28. $\frac{x^2-x-6}{x-3}=\frac{(x-3)(x+2)}{(x-3)}=x+2$

29. $\frac{t^2-4t-21}{t+3}=\frac{(t+3)(t-7)}{t+3}=t-7$

30. $\frac{2x-2x^2}{x^3-2x^2+x}=\frac{2x(1-x)}{x(x^2-2x+1)}$
$=\frac{-2x(x-1)}{x(x-1)(x-1)}$
$=-\frac{2}{x-1}$

31. $\frac{12}{x^2+2x}+\frac{4}{x}+\frac{2}{x+2}$
$=\frac{12}{x(x+2)}+\frac{4(x+2)}{x(x+2)}+\frac{2x}{x(x+2)}$
$=\frac{12+4x+8+2x}{x(x+2)}$
$=\frac{6x+20}{x(x+2)}$
$=\frac{2(3x+10)}{x(x+2)}$

32. $\frac{2}{6y-2}+\frac{y}{9y^2-1}-\frac{2y+1}{1-3y}$
$=\frac{2}{2(3y-1)}+\frac{y}{(3y+1)(3y-1)}+\frac{2y+1}{3y-1}$
$=\frac{2(3y+1)}{2(3y+1)(3y-1)}+\frac{2y}{2(3y+1)(3y-1)}$
$+\frac{(2y+1)(3y+1)}{2(3y+1)(3y-1)}$
$=\frac{6y+2+2y+12y^2+10y+2}{2(3y+1)(3y-1)}$

$$= \frac{12y^2+18y+4}{2(3y+1)(3y-1)}$$

$$= \frac{2(6y^2+9y+2)}{2(3y+1)(3y-1)}$$

$$= \frac{6y^2+9y+2}{(3y+1)(3y-1)}$$

33. $\frac{t^2+t-12}{x^2-1} \cdot \frac{x^2-6x-7}{8t-t^2-15}$

$$= \frac{(t+4)(t-3)(x-7)(x+1)}{-(x+1)(x-1)(t-3)(t-5)}$$

$$= -\frac{(t+4)(x-7)}{(x-1)(t-5)}$$

34. $\dfrac{\frac{x}{x-3}-\frac{2}{x^2-4x+3}}{\frac{5}{x-1}+\frac{5}{x+3}} = \dfrac{\frac{x}{x-3}-\frac{2}{(x-3)(x-1)}}{\frac{5}{x-1}+\frac{5}{x-3}}$

$$= \frac{x(x-1)-2}{5(x-3)+5(x-1)}$$

$$= \frac{x^2-x-2}{5x-15+5x-5}$$

$$= \frac{(x-2)(x+1)}{10(x-2)} = \frac{x+1}{10}$$

35. **a.** $0 \cdot 0 = 0$ **b.** $\frac{0}{0}$ is undefined.

c. $\frac{0}{17} = 0$ **d.** $\frac{3}{0}$ is undefined.

e. $0^5 = 0$ **f.** $17^0 = 1$

36. If $\frac{0}{0} = a$, then $0 = 0 \cdot a$, but this is meaningless because a could be any real number. No single value satisfies $\frac{0}{0} = a$.

37. **a.** $-3 < -7$; False **b.** $-1 > -17$; True

c. $-3 < -\frac{22}{7}; -\frac{21}{7} < -\frac{22}{7}$; False

d. $-5 > -\sqrt{26}; -\sqrt{25} > -\sqrt{26}$; True

e. $\frac{6}{7} < \frac{34}{39}; \frac{234}{273} < \frac{238}{273}$; True

f. $-\frac{5}{7} < -\frac{44}{59}; -\frac{295}{413} < -\frac{308}{413}$; False

38. **a.** $a < b; a^2 < ab$ and $ab < b^2$, so $a^2 < b^2$

b. $a < b;\ \frac{a}{b} < 1;\ \frac{1}{b} < \frac{1}{a};\ \frac{1}{a} > \frac{1}{b}$

39. $a < b$; $2a < a + b$ and $a + b < 2b$, so $2a < a + b < 2b;\ a < \frac{a+b}{2} < b$

40. **a.** is false if $a < 0$

b. is always true

c. is always true

d. is false

41. **a.** True; If x is positive, then x^2 is positive.

b. False; Take $x = -2$. Then $x^2 > 0$ but $x < 0$.

c. False; Take $x = \frac{1}{2}$. Then $x^2 = \frac{1}{4} < x$.

d. True; $n = 2$ is even and prime.

e. True; Let x be any number. Take $y = x^2 + 1$. Then $y > x^2$.

f. False; There is no number larger than every x^2.

g. True; $1/n$ can be made arbitrarily close to 0.

h. True 2^{-n} can be made arbitrarily close to 0.

42. **a.** If n is odd, then there is an integer k such that $n = 2k+1$. Then $n^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$

b. Prove the contrapositive. Suppose n is even. Then there is an integer k such that $n = 2k$. Then

$n^2 = (2k)^2 = 4k^2 = 2(2k^2)$

Thus n^2 is even.

c. Parts (a) and (b) prove that n is odd if and only if n^2 is odd.

43. a. $243 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

b. $127 = 1 \cdot 127$

c. $5100 = 2 \cdot 2550 = 2 \cdot 2 \cdot 1275 = 2 \cdot 2 \cdot 3 \cdot 425$
$= 2 \cdot 2 \cdot 3 \cdot 5 \cdot 85 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5 \cdot 17$

d. $346 = 2 \cdot 173$

44. Let $A = b \cdot c^2 \cdot d^3$; then $A^2 = b^2 \cdot c^4 \cdot d^6$, so the square of the number is the product of primes which occur an even number of times

45. $\sqrt{2} = \frac{p}{q}; 2 = \frac{p^2}{q^2}; 2q^2 = p^2$; Since the prime factors of p^2 must occur an even number of times, $2q^2$ would not be valid and $\frac{p}{q} = \sqrt{2}$ must be irrational.

46. $\sqrt{3} = \frac{p}{q}$; $3 = \frac{p^2}{q^2}$; $3q^2 = p^2$; Since the prime factors of p^2 must occur an even number of times, $3q^2$ would not be valid and $\frac{p}{q} = \sqrt{3}$ must be irrational.

47. Let a, b, p, and q be natural numbers, so $\frac{a}{b}$ and $\frac{p}{q}$ are rational. $\frac{a}{b} + \frac{p}{q} = \frac{aq + bp}{bq}$ This sum is the quotient of natural numbers, so it is also rational.

48. Let a be an irrational number and p and q be natural numbers. $a \cdot \frac{p}{q} = \frac{ap}{q}$. Since the numerator is not a natural number, the product is irrational.

49. a. $-\sqrt{9} = -3$; rational

b. $0.375 = \frac{3}{8}$; rational

c. $1 - \sqrt{2}$; irrational

d. $(1 + \sqrt{3})^2 = 1 + 2\sqrt{3} + 3 = 4 + 2\sqrt{3}$; irrational

e. $(3\sqrt{2})(5\sqrt{2}) = 15\sqrt{4} = 30$; rational

e. $5\sqrt{2}$; irrational

50. The sum of two irrational numbers is not always irrational. If the numbers are additive inverses, the sum is 0, which is rational.

51. If m were a perfect square, its prime factors would occur even numbers of times. If m is not a perfect square, some factors will occur an odd number of times and $\sqrt{m}$ will be irrational.

52. $\sqrt{6} + \sqrt{3} \approx 4.18154$ is irrational.

53. $\sqrt{2} - \sqrt{3} + \sqrt{6} \approx 2.13165$ is irrational.

54. $\log_{10} 5 \approx 0.69897$ is irrational

55. a. Converse: If I get an A in this course, then I do all of the homework.

Contrapositive: If I don't get an A in this course, then I don't do all of the homework.

b. Converse: If x is an integer, then x is a real number.

Contrapositive: If x is not an integer, then x is not a real number.

c. Converse: If ΔABC is an isosceles triangle, then ΔABC is an equilateral triangle.

Contrapositive: If ΔABC is not an isosceles triangle, then ΔABC is not an equilateral triangle.

1.2 Concepts Review

1. 0.333..... (3s repeat); 0.200... (0s repeat); 3.14159...

2. rational

3. rational; irrational

4. real

Problem Set 1.2

1. $$\begin{array}{r} .0833 \\ 12\overline{)1.0000} \\ \underline{96} \\ 40 \\ \underline{36} \\ 40 \end{array}$$

2. $$\begin{array}{r} .285714 \\ 7\overline{)2.000000} \\ \underline{14} \\ 60 \\ \underline{56} \\ 40 \\ \underline{35} \\ 50 \\ \underline{49} \\ 10 \\ \underline{7} \\ 30 \\ \underline{28} \\ 2 \end{array}$$

3. $$\begin{array}{r} .846153 \\ 13\overline{)11.000000} \\ \underline{104} \\ 60 \\ \underline{52} \\ 80 \\ \underline{78} \\ 20 \\ \underline{13} \\ 70 \\ \underline{65} \\ 50 \\ \underline{39} \\ 11 \end{array}$$

4. $$\begin{array}{r} .294117 \\ 17\overline{)5.000000} \\ \underline{34} \\ 160 \\ \underline{153} \\ 70 \\ \underline{68} \\ 20 \\ \underline{17} \\ 30 \\ \underline{17} \\ 130 \\ \underline{119} \\ 11 \end{array}$$

5. $$\begin{array}{r} 3.66 \\ 3\overline{)11.00} \\ \underline{9} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ 2 \end{array}$$

6. $$\begin{array}{r} .846153 \\ 13\overline{)11.000000} \\ \underline{10\ 4} \\ 60 \\ \underline{52} \\ 80 \\ \underline{78} \\ 20 \\ \underline{13} \\ 70 \\ \underline{65} \\ 50 \\ \underline{39} \\ 11 \end{array}$$

7. $$\begin{aligned} x &= 0.123123123... \\ 1000x &= 123.123123... \\ x &= 0.123123... \\ \hline 999x &= 123 \\ x &= \frac{123}{999} = \frac{41}{333} \end{aligned}$$

8. $$\begin{aligned} x &= 0.217171717... \\ 1000x &= 217.171717... \\ 10x &= 2.171717... \\ \hline 990x &= 215 \\ x &= \frac{215}{990} = \frac{43}{198} \end{aligned}$$

9. $$\begin{aligned} x &= 2.56565656... \\ 100x &= 256.565656... \\ x &= 2.565656... \\ \hline 99x &= 254 \\ x &= \frac{254}{99} \end{aligned}$$

10. $$\begin{aligned} x &= 3.929292... \\ 100x &= 392.929292... \\ x &= 3.929292... \\ \hline 99x &= 389 \\ x &= \frac{389}{99} \end{aligned}$$

11. $$\begin{aligned} x &= 0.199999... \\ 100x &= 19.99999... \\ 10x &= 1.99999... \\ \hline 90x &= 18 \\ x &= \frac{18}{90} = \frac{1}{5} \end{aligned}$$

12. $$\begin{aligned} x &= 0.399999... \\ 100x &= 39.99999... \\ 10x &= 3.99999... \\ \hline 90x &= 36 \\ x &= \frac{36}{90} = \frac{2}{5} \end{aligned}$$

13. Those rational numbers that can be expressed by a terminating decimal followed by zeros.

14. $\frac{p}{q} = p\left(\frac{1}{q}\right)$, so we only need to look at $\frac{1}{q}$. If $q = 2^n \cdot 5^m$, then $\frac{1}{q} = \left(\frac{1}{2}\right)^n \cdot \left(\frac{1}{5}\right)^m = (0.5)^n (0.2)^m$. The product of any number of terminating decimals is also a terminating decimal, so $(0.5)^n$ and $(0.2)^m$, and hence their product, $\frac{1}{q}$, is a terminating decimal. Thus $\frac{p}{q}$ has a terminating decimal expansion.

15. Answers will vary. Possible answer: 0.000001, $\frac{1}{\pi^{12}} \approx 0.0000010819...$

16. Smallest positive integer: 1; There is no smallest positive rational or irrational number.

17. Answers will vary. Possible answer: 3.14159101001...

18. There is no real number between 0.9999... (repeating 9's) and 1. 0.9999... and 1 represent the *same* real number.

19. Irrational

20. Answers will vary. Possible answers: $-\pi$ and π, $-\sqrt{2}$ and $\sqrt{2}$

21. $(\sqrt{3}+1)^3 \approx 20.39230485$

22. $\left(\sqrt{2}-\sqrt{3}\right)^4 \approx 0.0102051443$

23. $\sqrt[4]{1.123}-\sqrt[3]{1.09} \approx 0.00028307388$

24. $(3.1415)^{-1/2} \approx 0.5641979034$

25. $\dfrac{\sqrt{130}-\sqrt{5}}{3^{1.2}-3} \approx 12.43322783$

26. $\dfrac{(0.00121)\left(5.23\times 10^{-3}\right)}{6.16\times 10^{-4}} \approx 0.0102732143$

27. $\sqrt{8.9\pi^2+1}-3\pi \approx 0.000691744752$

28. $\sqrt[4]{(6\pi^2-2)\pi} \approx 3.661591807$

29. Let a and b be real numbers with $a < b$. Let n be a natural number that satisfies $1/n < b-a$. Let $S = \{k : k/n > b\}$. Since a nonempty set of integers that is bounded below contains a least element, there is a $k_0 \in S$ such that $k_0/n > b$ but $(k_0-1)/n \le b$. Then
$$\frac{k_0-1}{n} = \frac{k_0}{n} - \frac{1}{n} > b - \frac{1}{n} > a$$
Thus, $a < \frac{k_0-1}{n} \le b$. If $\frac{k_0-1}{n} < b$, then choose $r = \frac{k_0-1}{n}$. Otherwise, choose $r = \frac{k_0-2}{n}$.

30. Answers will vary. Possible answer: $\approx 120 \text{ in}^3$

31. $r = 4000 \text{ mi} \times 5280\dfrac{\text{ft}}{\text{mi}} = 21,120,000 \text{ ft}$

equator $= 2\pi r = 2\pi(21,120,000) \approx 132,700,874$ ft

32. Answers will vary. Possible answer:
$$70\frac{\text{beats}}{\text{min}} \times 60\frac{\text{min}}{\text{hr}} \times 24\frac{\text{hr}}{\text{day}} \times 365\frac{\text{day}}{\text{year}} \times 20 \text{ yr}$$
$= 735,840,000$ beats

33. $V = \pi r^2 h = \pi\left(\dfrac{16}{2}\cdot 12\right)^2 (270\cdot 12)$

$\approx 93,807,453.98 \text{ in.}^3$

volume of one board foot (in inches):
1 3 12 3 12 = 144 in.^{3}
number of board feet:
$$\frac{93,807,453.98}{144} \approx 651,441 \text{ board ft}$$

34. $V = \pi(8.004)^2(270) - \pi(8)^2(270) \approx 54.3 \text{ in.}^3$

35. a. At $x = 2\pi$: 286.866542

b. At $x = 2.15$: 9.16925

c. At $x = 2.71828$: 16.34874967

d. At $x = 1.1$: 4.292

36. $x^4 - 3x^3 + 4x^2 + 6x - 10$
$= (x^3 - 3x^2 + 4x + 6)x - 10$
$= \left[(x^2 - 3x + 4)x + 6\right]x - 10$
$= \left[((x-3)x + 4)x + 6\right]x - 10$

a. At $x = 1; -2$

b. At $x = \pi; 52.71823452$

c. At $x = 14.2; 32,950.5856$

d. At $x = 1.2157; 0.0000269681$

37. a. -2 b. -2

c. $x = 2.4444...;$
$10x = 24.4444...$
$x = 2.4444...$
$9x = 22$
$x = \dfrac{22}{9}$

d. 1

e. $n = 1: x = 0,\ n = 2:\ x = \dfrac{3}{2},\ n = 3:\ x = -\dfrac{2}{3},$
$n = 4:\ x = \dfrac{5}{4}$

The upper bound is $\dfrac{3}{2}$.

f. $\sqrt{2}$

38. a. Answers will vary. Possible answer: An example is $S = \{x : x^2 < 5, x \text{ a rational number}\}$. Here the least upper bound is $\sqrt{5}$, which is real but irrational.

b. True

1.3 Concepts Review

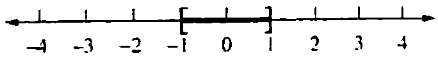
1. interval; intervals

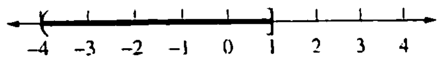
2. $[-1,5); (-\infty, 2]$

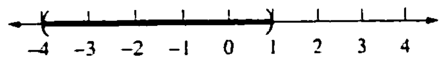
3. $b > 0; b < 0$

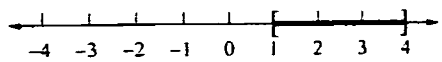
4. $-5, -4, 3$

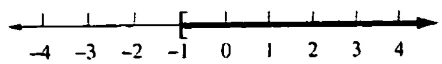
Problem Set 1.3

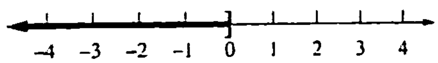
1. a. 

   b. 

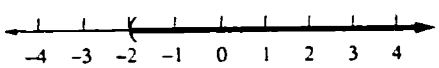
   c. 

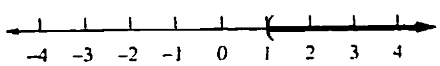
   d. 

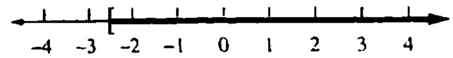
   e. 

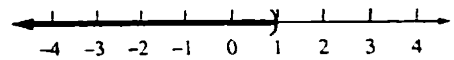
   f. 

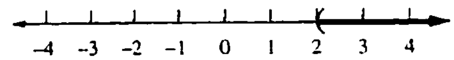
2. a. $(2,7)$ b. $[-3,4)$
   c. $(-\infty, 2]$ d. $[-1,3]$

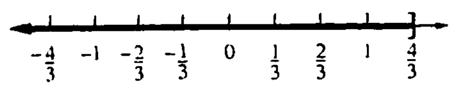
3. $x - 7 < 2x - 5$
   $-2 < x; (-2, \infty)$


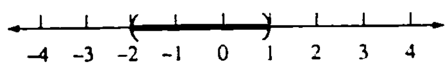
4. $3x - 5 < 4x - 6$
   $1 < x; (1, \infty)$


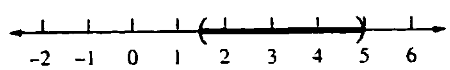
5. $7x - 2 \le 9x + 3$
   $-5 \le 2x$
   $x \ge -\frac{5}{2}; \left[-\frac{5}{2}, \infty\right)$


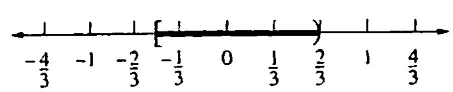
6. $5x - 3 > 6x - 4$
   $1 > x; (-\infty, 1)$


7. $10x + 1 > 8x + 5$
   $2x > 4$
   $x > 2; (2, \infty)$


8. $-2x + 5 \ge 4x - 3$
   $8 > 6x$
   $x \le \frac{4}{3}; \left(-\infty, \frac{4}{3}\right)$


9. $-4 < 3x + 2 < 5$
   $-6 < 3x < 3$
   $-2 < x < 1; (-2, -1)$


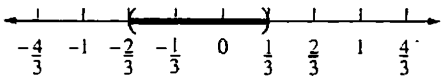
10. $-3 < 4x - 9 < 11$
    $6 < 4x < 20$
    $\frac{3}{2} < x < 5; \left(\frac{3}{2}, 5\right)$


11. $-3 < 1 - 6x \le 4$
    $-4 < -6x \le 3$
    $\frac{2}{3} > x \ge -\frac{1}{2}; \left[-\frac{1}{2}, \frac{2}{3}\right)$


12. $4 < 5 - 3x < 7$

$-1 < -3x < 2$

$\frac{1}{3} > x > -\frac{2}{3};\ \left(-\frac{2}{3}, \frac{1}{3}\right)$

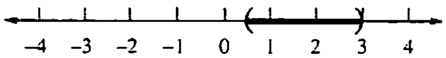


13. $2 + 3x < 5x + 1 < 16$

$2 + 3x < 5x + 1$ and $5x + 1 < 16$

$1 < 2x$ and $5x < 15$

$x > \frac{1}{2}$ and $x < 3;\ \left(\frac{1}{2}, 3\right)$

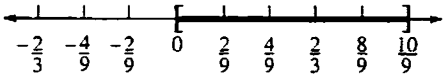


14. $2x - 4 \le 6 - 7x \le x + 6$

$2x - 4 \le -7x$ and $6 - 7x \le 3x + 6$

$9x \le 10$ and $10x \ge 0$

$x \le \frac{10}{9}$ and $x \ge 0;\left[0, \frac{10}{9}\right]$



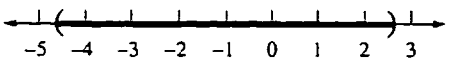
15. $x^2 + 2x - 12 < 0;$

$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-12)}}{2(1)} = \frac{-2 \pm \sqrt{52}}{2}$

$= -1 \pm \sqrt{13}$

$\left[x - \left(-1 + \sqrt{13}\right)\right]\left[x - \left(-1 - \sqrt{13}\right)\right] < 0;$

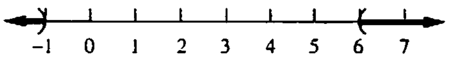
$\left(-1 - \sqrt{13}, -1 + \sqrt{13}\right)$



16. $x^2 - 5x - 6 > 0$

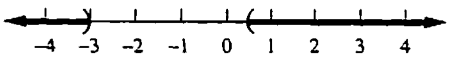
$(x + 1)(x - 6) > 0;$

$(-\infty, 1) \cup (6, \infty)$



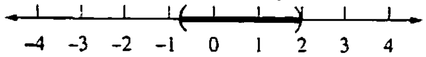
17. $2x^2 + 5x - 3 > 0;\ (2x - 1)(x + 3) > 0;$

$(-\infty, -3) \cup \left(\frac{1}{2}, \infty\right)$

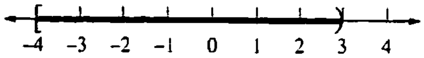


18. $4x^2 - 5x - 6 < 0$

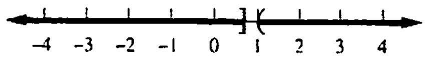
$(4x + 3)(x - 2) < 0;\left(-\frac{3}{4}, 2\right)$



19. $\frac{x + 4}{x - 3} \le 0;\ [-4, 3)$



20. $\frac{3x - 2}{x - 1} \ge 0;\left(-\infty, \frac{2}{3}\right] \cup (1, \infty)$

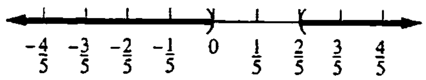


21. $\frac{2}{x} < 5$

$\frac{2}{x} - 5 < 0$

$\frac{2 - 5x}{x} < 0;$

$(-\infty, 0) \cup \left(\frac{2}{5}, \infty\right)$

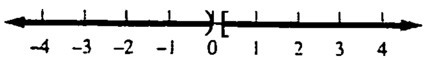


22. $\frac{7}{4x} \le 7$

$\frac{7}{4x} - 7 \le 0$

$\frac{7 - 28x}{4x} \le 0;$

$(-\infty, 0) \cup \left[\frac{1}{4}, \infty\right)$

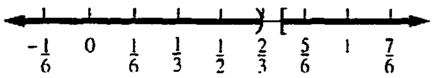


23. $$\frac{1}{3x-2} \le 4$$

$$\frac{1}{3x-2} - 4 \le 0$$

$$\frac{1-4(3x-2)}{3x-2} \le 0$$

$$\frac{9-12x}{3x-2} \le 0; \left(-\infty, \frac{2}{3}\right) \cup \left[\frac{3}{4}, \infty\right)$$

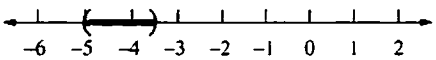


24. $$\frac{3}{x+5} > 2$$

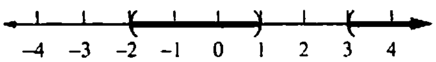
$$\frac{3}{x+5} - 2 > 0$$

$$\frac{3-2(x+5)}{x+5} > 0$$

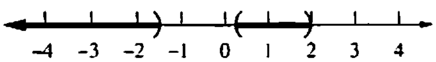
$$\frac{-2x-7}{x+5} > 0; \left(-5, -\frac{7}{2}\right)$$



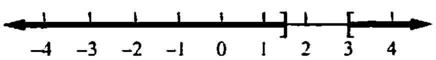
25. $(x+2)(x-1)(x-3) > 0; (-2,1) \cup (3,8)$



26. $(2x+3)(3x-1)(x-2) < 0; \left(-\infty, -\frac{3}{2}\right) \cup \left(\frac{1}{3}, 2\right)$

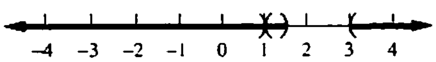


27. $(2x-3)(x-1)^2(x-3) \ge 0; \left(-\infty, \frac{3}{2}\right] \cup [3, \infty)$



28. $(2x-3)(x-1)^2(x-3) > 0;$

$(-\infty, 1) \cup \left(1, \frac{3}{2}\right) \cup (3, \infty)$

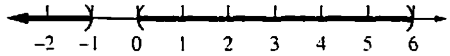


29. $x^3 - 5x^2 - 6x < 0$

$x(x^2 - 5x - 6) < 0$

$x(x+1)(x-6) < 0;$

$(-\infty, -1) \cup (0, 6)$

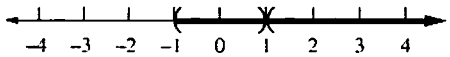


30. $x^3 - x^2 - x + 1 > 0$

$(x^2 - 1)(x - 1) > 0$

$(x+1)(x-1)^2 > 0;$

$(-1, 1) \cup (1, \infty)$



31. a. $3x + 7 > 1$ and $2x + 1 < 3$
$3x > -6$ and $2x < 2$
$x > -2$ and $x < 1$; $(-2, 1)$

b. $3x + 7 > 1$ and $2x + 1 > -4$
$3x > -6$ and $2x > -5$
$x > -2$ and $x > -\frac{5}{2}$; $(-2, \infty)$

c. $3x + 7 > 1$ and $2x + 1 < -4$
$x > -2$ and $x < -\frac{5}{2}$; $\varnothing$

32. a. $2x - 7 > 1$
$\{4 < x\}$ or $2x + 1 < 3$
$2x > 8$ or $2x < 2$; $x > 4$ or $x < 1$;
$(-\infty, 1) \cup (4, \infty)$

b. $2x - 7 \le 1$
$\{x \le 4\}$ or $2x + 1 < 3$
$x \le 4$ or $x < 1$; $(-\infty, 4]$

c. $2x - 7 \le 1$
$\{x \le 4\}$ or $2x + 1 > 3$
$x \le 4$ or $x > 1$; $(-\infty, \infty)$

33. a. $(x+1)(x^2 + 2x - 7) \ge x^2 - 1$
$x^3 + 3x^2 - 5x - 7 \ge x^2 - 1$
$x^3 + 2x^2 - 5x - 6 \ge 0$
$(x+3)(x+1)(x-2) \ge 0$
$[-3, -1] \cup [2, \infty)$

b. $x^4 - 2x^2 \ge 8$

$x^4 - 2x^2 - 8 \ge 0$

$(x^2 - 4)(x^2 + 2) \ge 0$

$(x^2 + 2)(x + 2)(x - 2) \ge 0$

$(-\infty, 12] \cup [2, \infty)$

c. $(x^2 + 1)^2 - 7(x^2 + 1) + 10 < 0$

$[(x^2 + 1) - 5][(x^2 + 1) - 2] < 0$

$(x^2 - 4)(x^2 - 1) < 0$

$(x + 2)(x + 1)(x - 1)(x - 2) < 0$

$(-2, -1) \cup (1, 2)$

34. Suppose $x > 0$. If we divide both sides of the inequality $1 > 0$ by x, we obtain $1/x > 0$. To prove the converse, divide both sides of the equation $1 > 0$ by $1/x$. This gives $\frac{1}{1/x} > \frac{0}{1/x}$, which is equivalent to $x > 0$.

35. **a.** $1.99 < \frac{1}{x} < 2.01$

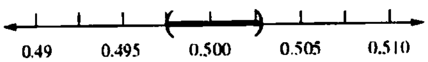
$1.99x < 1 < 2.01x$

$1.99x < 1$ and $1 < 2.01x$

$x < \frac{1}{1.99}$ and $x > \frac{1}{2.01}$

$\frac{1}{2.01} < x < \frac{1}{1.99}$

$\left(\frac{1}{2.01}, \frac{1}{1.99}\right)$



b. $2.99 < \frac{1}{x+2} < 3.01$

$2.99(x + 2) < 1 < 3.01(x + 2)$

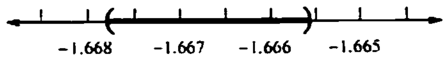
$2.99x + 5.98 < 1$ and

$1 < 3.01x + 6.02$

$x < \frac{-4.98}{2.99}$ and $x > \frac{-5.02}{3.01}$

$-\frac{5.02}{3.01} < x < -\frac{4.98}{2.99}$

$\left(-\frac{5.02}{3.01}, -\frac{4.98}{2.99}\right)$



c. $3 - \varepsilon < \frac{1}{x+2} < 3 + \varepsilon$

$(3 - \varepsilon)(x + 2) < 1 < (3 + \varepsilon)(x + 2)$

$(3 - \varepsilon)x + (3 - \varepsilon)2 < 1 < (3 + \varepsilon)x + (3 + \varepsilon)2$

$x < \frac{1 - 2(3 - \varepsilon)}{3 - \varepsilon}$ and $x > \frac{1 - 2(3 + \varepsilon)}{3 + \varepsilon}$

$\left(\frac{1 - 2(3 + \varepsilon)}{3 + \varepsilon}, \frac{1 - 2(3 - \varepsilon)}{3 - \varepsilon}\right)$

36. $1 + x + x^2 + x^3 + \ldots + x^{99} \le 0$;

$(-\infty, -1)$

37. $\frac{1}{R} \le \frac{1}{10} + \frac{1}{20} + \frac{1}{30}$

$\frac{1}{R} \le \frac{6 + 3 + 2}{60}$

$\frac{1}{R} \le \frac{11}{60}$

$R \ge \frac{60}{11}$

$\frac{1}{R} \ge \frac{1}{20} + \frac{1}{30} + \frac{1}{40}$

$\frac{1}{R} \ge \frac{6 + 4 + 3}{120}$

$\frac{1}{R} \ge \frac{13}{120}$

$R \le \frac{120}{13}$

$\frac{60}{11} \le R \le \frac{120}{13}$

1.4 Concepts Review

1. $-1; 5$

2. $|a+b| \le |a|+|b|$

3. b, c

4. $\frac{0.2}{5} = 0.04$

Problem Set 1.4

1. $|x+2| < 1;$
   $-1 < x+2 < 1$
   $-3 < x < -1$
   $(-3, -1)$

2. $|x-2| \ge 5;$
   $x-2 \le -5$ or $x-2 \ge 5$
   $x \le -3$ or $x \ge 7$
   $(-\infty, -3] \cup [7, \infty)$

3. $|2x-1| > 2;$
   $2x - 1 < -2$ or $2x - 1 > 2$
   $2x < -1$ or $2x > 3;$
   $x < -\frac{1}{2}$ or $x > \frac{3}{2}; \left(-\infty, -\frac{1}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$

4. $|4x+5| \le 10;$
   $-10 \le 4x+5 \le 10$
   $-15 \le 4x \le 5$
   $-\frac{15}{4} \le x \le \frac{5}{4}; \left[-\frac{15}{4}, \frac{5}{4}\right]$

5. $\left|\frac{x}{4}+1\right| < 1$
   $-1 < \frac{x}{4}+1 < 1$
   $-2 < \frac{x}{4} < 0;$
   $-8 < x < 0; (-8, 0)$

6. $\left|\frac{2x}{7}-5\right| \ge 7$
   $\frac{2x}{7}-5 \le -7$ or $\frac{2x}{7}-5 \ge 7$
   $\frac{2x}{7} \le -2$ or $\frac{2x}{7} \ge 12$
   $x \le -7$ or $x \ge 42;$
   $(-\infty, -7] \cup [42, \infty)$

7. $|2x-7| > 3;$
   $2x - 7 < -3$ or $2x - 7 > 3$
   $2x < 4$ or $2x > 10$
   $x < 2$ or $x > 5$; $(-\infty, 2) \cup (5, \infty)$

8. $|5x-6| > 1;$
   $5x-6 < -1$ or $5x-6 > 1$
   $5x < 5$ or $5x > 7$
   $x < 1$ or $x > \frac{7}{5}; (-\infty, 1) \cup \left(\frac{7}{5}, \infty\right)$

9. $|4x+2| \ge 10;$
   $4x + 2 \le -10$ or $4x + 2 \ge 10$
   $4x \le -12$ or $4x \ge 8$
   $x \le -3$ or $x \ge 2$
   $(-\infty, -3] \cup [2, \infty)$

10. $\left|\frac{x}{2}+7\right| \ge 2;$
    $\frac{x}{2}+7 \le -2$ or $\frac{x}{2}+7 \ge 2$
    $\frac{x}{2} \le -9$ or $\frac{x}{2} \ge -5$
    $x \le -18$ or $x \ge -10$
    $(-\infty, -18] \cup [-10, \infty)$

11. $\left|2+\frac{5}{x}\right| > 1;$
    $2+\frac{5}{x} < -1$ or $2+\frac{5}{x} > 1$
    $3+\frac{5}{x} < 0$ or $1+\frac{5}{x} > 0$
    $\frac{3x+5}{x} < 0$ or $\frac{x+5}{x} > 0;$
    $(-\infty, -5) \cup \left(-\frac{5}{3}, 0\right) \cup (0, \infty)$

12. $\left|\frac{1}{x}-3\right| > 6;$
    $\frac{1}{x}-3 < -6$ or $\frac{1}{x}-3 > 6$
    $\frac{1}{x}+3 < 0$ or $\frac{1}{x}-9 > 0$
    $\frac{1+3x}{x} < 0$ or $\frac{1-9x}{x} > 0;$
    $\left(\frac{-1}{3}, 0\right) \cup \left(0, \frac{1}{9}\right)$

13. $x^2-3x-4\ge 0;$

$$x=\frac{3\pm\sqrt{(-3)^2-4(1)(-4)}}{2(1)}=\frac{3\pm 5}{2}=-1,4$$

$(x+1)(x-4)=0;(-\infty,-1]\cup[4,\infty)$

14. $x^2-4x+4\le 0;\ x=\frac{4\pm\sqrt{(-4)^2-4(1)(4)}}{2(1)}=2$

$(x-2)(x-2)\le 0;x=2$

15. $3x^2+17x-6>0;$

$$x=\frac{-17\pm\sqrt{(17)^2-4(3)(-6)}}{2(3)}=\frac{-17\pm 19}{6}=-6,\frac{1}{3}$$

$(3x-1)(x+6)>0;\ (-\infty,-6)\cup\left(\frac{1}{3},\infty\right)$

16. $14x^2+11x-15\le 0;$

$$x=\frac{-11\pm\sqrt{(11)^2-4(14)(-15)}}{2(14)}=\frac{-11\pm 31}{28}$$

$x=-\frac{3}{2},\frac{5}{7}$

$\left(x+\frac{3}{2}\right)\left(x-\frac{5}{7}\right)\le 0;\left[-\frac{3}{2},\frac{5}{7}\right]$

17. $|x-3|<0.5\Leftrightarrow 5|x-3|<5(0.5)\Leftrightarrow|5x-15|<2.5$

18. $|x+2|<0.3\Leftrightarrow 4|x+2|<4(0.3)\Leftrightarrow|4x+18|<1.2$

19. $|x-2|<\frac{\varepsilon}{6}\Leftrightarrow 6|x-2|<\varepsilon\Leftrightarrow|6x-12|<\varepsilon$

20. $|x+4|<\frac{\varepsilon}{2}\Leftrightarrow 2|x+4|<\varepsilon\Leftrightarrow|2x+8|<\varepsilon$

21. $|3x-15|<\varepsilon\Leftrightarrow|3(x-5)|<\varepsilon$

$\Leftrightarrow 3|x-5|<\varepsilon$

$\Leftrightarrow|x-5|<\frac{\varepsilon}{3};\delta=\frac{\varepsilon}{3}$

22. $|4x-8|<\varepsilon\Leftrightarrow|4(x-2)|<\varepsilon$

$\Leftrightarrow 4|x-2|<\varepsilon$

$\Leftrightarrow|x-2|<\frac{\varepsilon}{4};\delta=\frac{\varepsilon}{4}$

23. $|6x+36|<\varepsilon\Leftrightarrow|6(x+6)|<\varepsilon$

$\Leftrightarrow 6|x+6|<\varepsilon$

$\Leftrightarrow|x+6|<\frac{\varepsilon}{6};\delta=\frac{\varepsilon}{6}$

24. $|5x+25|<\varepsilon\Leftrightarrow|5(x+5)|<\varepsilon$

$\Leftrightarrow 5|x+5|<\varepsilon$

$\Leftrightarrow|x+5|<\frac{\varepsilon}{5};\delta=\frac{\varepsilon}{5}$

25. $C=\pi d$

$|C-10|\le 0.02$

$|\pi d-10|\le 0.02$

$\left|\pi\left(d-\frac{10}{\pi}\right)\right|\le 0.02$

$\left|d-\frac{10}{\pi}\right|\le\frac{0.02}{\pi}\approx 0.0064$

We must measure the diameter to an accuracy of 0.0064 in.

26. $|C-50|\le 1.5,\left|\frac{5}{9}(F-32)-50\right|\le 1.5;$

$\frac{5}{9}|(F-32)-90|\le 1.5$

$|F-122|\le 2.7$

We are allowed an error of 2.7° F.

27. $|x-1|<2|x-3|$

$|x-1|<|2x-6|$

$(x-1)^2<(2x-6)^2$

$x^2-2x+1<4x^2-24x+36$

$3x^2-22x+35>0$

$(3x-7)(x-5)>0;$

$\left(-\infty,\frac{7}{3}\right)\cup(5,\infty)$

28. $|2x-1|\ge|x+1|$

$(2x-1)^2\ge(x+1)^2$

$4x^2-4x+1\ge x^2+2x+1$

$3x^2-6x\ge 0$

$3x(x-2)\ge 0$

$(-\infty,0]\cup[2,\infty)$

29.
$$2|2x-3|<|x+10|$$
$$|4x-6|<|x+10|$$
$$(4x-6)^2<(x+10)^2$$
$$16x^2-48x+36<x^2+20x+100$$
$$15x^2-68x-64<0$$
$$(5x+4)(3x-16)<0:$$
$$\left(-\frac{4}{5},\frac{16}{3}\right)$$

30.
$$|3x-1|<2|x+6|$$
$$|3x-1|<|2x+12|$$
$$(3x-1)^2<(2x-12)^2$$
$$9x^2-6x+1<4x^2+48x+144$$
$$5x^2-54x-143<0$$
$$(5x+11)(x-13)<0;$$
$$\left(-\frac{11}{5},13\right)$$

31. $|x|<|y| \Rightarrow |x||x| \le |x||y|$ and $|x||y|<|y||y|$ Order property: $x<y \Leftrightarrow xz<yz$ when z is positive.

$\Rightarrow |x|^2<|y|^2$ Transitivity

$\Rightarrow x^2<y^2$ $\left(|x|^2=x^2\right)$

Conversely,

$x^2<y^2 \Rightarrow |x|^2<|y|^2$ $\left(x^2=|x|^2\right)$

$\Rightarrow |x|^2-|y|^2<0$ Subtract $|y|^2$ from each side.

$\Rightarrow (|x|-|y|)(|x|+|y|)<0$ Factor the difference of two squares.

$\Rightarrow |x|-|y|<0$ This is the only factor that can be negative.

$\Rightarrow |x|<|y|$ Add $|y|$ to each side.

32. $0<a<b \Rightarrow a=\left(\sqrt{a}\right)^2$ *and* $b=\left(\sqrt{b}\right)^2$, so $\left(\sqrt{a}\right)^2<\left(\sqrt{b}\right)^2$, and, by Problem 31, $\left|\sqrt{a}\right|<\left|\sqrt{b}\right| \Rightarrow \sqrt{a}<\sqrt{b}$

33. **a.** $|a-b|=|a+(-b)|\le|a|+|-b|=|a|+|b|$

b. $|a-b|\ge\big||a|-|b|\big|\ge|a|-|b|$ Use Property 4 of absolute values.

c. $|a+b+c|=|(a+b)+c|\le|a+b|+|c|$
$\le|a|+|b|+|c|$

34.
$$\left|\frac{1}{x^2+3}-\frac{1}{|x|+2}\right|=\left|\frac{1}{x^2+3}+\left(-\frac{1}{|x|+2}\right)\right|$$
$$\le\left|\frac{1}{x^2+3}\right|+\left|-\frac{1}{|x|+2}\right|$$
$$=\left|\frac{1}{x^2+3}\right|+\left|\frac{1}{|x|+2}\right|$$
$$=\frac{1}{x^2+3}+\frac{1}{|x|+2}$$

by the Triangular Inequality, and since $x^2+3>0,\ |x|+2>0 \Rightarrow \frac{1}{x^2+3}>0, \frac{1}{|x|+2}>0.$

$x^2+3\ge3$ and $|x|+2\ge2$, so $\frac{1}{x^2+3}\le\frac{1}{3}$ and $\frac{1}{|x|+2}\le\frac{1}{2}$, thus,

$$\frac{1}{x^2+3}+\frac{1}{|x|+2}\le\frac{1}{3}+\frac{1}{2}$$

35. $\left|\frac{x-2}{x^2+9}\right| = \left|\frac{x+(-2)}{x^2+9}\right|$

$\left|\frac{x-2}{x^2+9}\right| \le \left|\frac{x}{x^2+9}\right| + \left|\frac{-2}{x^2+9}\right|$

$\left|\frac{x-2}{x^2+9}\right| \le \frac{|x|}{x^2+9} + \frac{2}{x^2+9} = \frac{|x|+2}{x^2+9}$

Since $x^2+9 \ge 9, \frac{1}{x^2+9} \le \frac{1}{9}$

$\frac{|x|+2}{x^2+9} \le \frac{|x|+2}{9}$

$\left|\frac{x-2}{x^2+9}\right| \le \frac{|x|+2}{9}$

36. $|x| \le 2 \Rightarrow \left|x^2+2x+7\right| \le \left|x^2\right| + |2x| + 7$

$\le 4+4+7 = 15$

and $\left|x^2+1\right| \ge 1$ so $\frac{1}{x^2+1} \le 1$.

Thus, $\left|\frac{x^2+2x+7}{x^2+1}\right| = \left|x^2+2x+7\right|\left|\frac{1}{x^2+1}\right|$

$\le 15 \cdot 1 = 15$

37. $\left|x^4+\frac{1}{2}x^3+\frac{1}{4}x^2+\frac{1}{8}x+\frac{1}{16}\right|$

$\le \left|x^4\right| + \frac{1}{2}\left|x^3\right| + \frac{1}{4}\left|x^2\right| + \frac{1}{8}|x| + \frac{1}{16}$

$\le 1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}$ since $|x| \le 1$.

So $\left|x^4+\frac{1}{2}x^3+\frac{1}{4}x^2+\frac{1}{8}x+\frac{1}{16}\right| \le 1.9375 < 2$.

38. a. $x < x^2$

$x - x^2 < 0$

$x(1-x) < 0$

$x < 0$ or $x > 1$

b. $x^2 < x$

$x^2 - x < 0$

$x(x-1) < 0$

$0 < x < 1$

39. $a \ne 0 \Rightarrow$

$0 \le \left(a-\frac{1}{a}\right)^2 = a^2 - 2 + \frac{1}{a^2}$

so, $2 \le a^2 + \frac{1}{a^2}$ or $a^2 + \frac{1}{a^2} \ge 2$

40. $a < b$

$a+a < a+b$ and $a+b < b+b$

$2a < a+b < 2b$

$a < \frac{a+b}{2} < b$

41. $0 < a < b$

$a^2 < ab$ and $ab < b^2$

$a^2 < ab < b^2$

$a < \sqrt{ab} < b$

42. $\sqrt{ab} \le \frac{1}{2}(a+b) \Leftrightarrow ab \le \frac{1}{4}\left(a^2+2ab+b^2\right)$

$\Leftrightarrow 0 \le \frac{1}{4}a^2 - \frac{1}{2}ab + \frac{1}{4}b^2 = \frac{1}{4}\left(a^2-2ab+b^2\right)$

$\Leftrightarrow 0 \le \frac{1}{4}(a-b)^2$ which is always true.

43. For a rectangle the area is ab, while for a square the area is $a^2 = \left(\frac{a+b}{2}\right)^2$. From Problem 42, $\sqrt{ab} \le \frac{1}{2}(a+b) \Leftrightarrow ab \le \left(\frac{a+b}{2}\right)^2$ so the square has the largest area.

44. $A = \pi r^2; A = 4\pi(10)^2 = 400\pi$

$\left|4\pi r^2 - 400\pi\right| < 0.01$

$4\pi\left|r^2 - 100\right| < 0.01$

$\left|r^2 - 100\right| < \frac{0.01}{4\pi}$

$-\frac{0.01}{4\pi} < r^2 - 100 < \frac{0.01}{4\pi}$

$\sqrt{100 - \frac{0.01}{4\pi}} < r < \sqrt{100 + \frac{0.01}{4\pi}}$

$\delta \approx 0.00004$ in

1.5 Concepts Review

1. II; IV

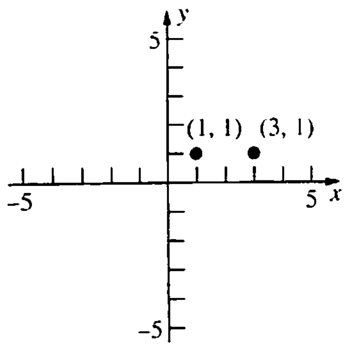
2. $\sqrt{(x+2)^2+(y-3)^2}$

3. $(x+4)^2+(y-2)^2=25$

4. $\left(\frac{-2+5}{2},\frac{3+7}{2}\right)=(1.5,5)$

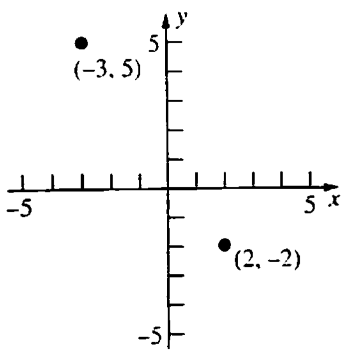
Problem Set 1.5

1.



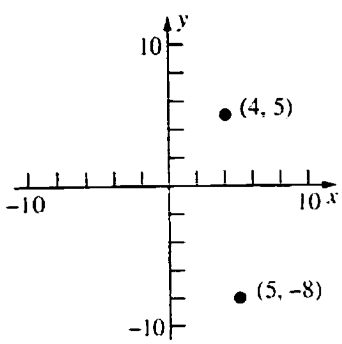
$d=\sqrt{(3-1)^2+(1-1)^2}=\sqrt{4}=2$

2.



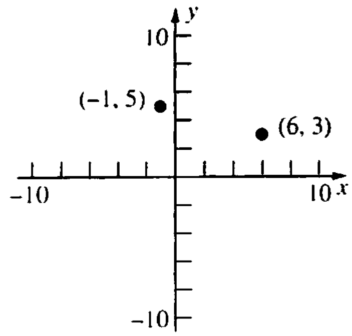
$d=\sqrt{(-3-2)^2+(5+2)^2}=\sqrt{74}\approx 8.60$

3.



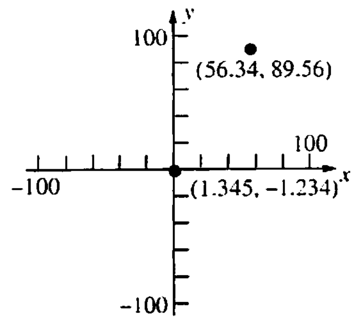
$d=\sqrt{(4-5)^2+(5+8)^2}=\sqrt{170}\approx 13.04$

4.



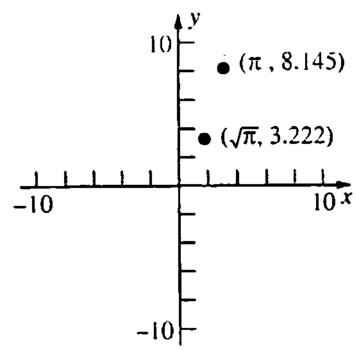
$d=\sqrt{(-1-6)^2+(5-3)^2}=\sqrt{49+4}=\sqrt{53}$
≈ 7.28

5.



$d=\sqrt{(1.345-56.34)^2+(-1.234-89.56)^2}$
≈ 106.151

6.



$d=\sqrt{(\sqrt{\pi}-\pi)^2+(3.222-8.145)^2}\approx 5.110$

7. $d_1=\sqrt{(5+2)^2+(3-4)^2}=\sqrt{49+1}=\sqrt{50}$
$d_2=\sqrt{(5-10)^2+(3-8)^2}=\sqrt{25+25}=\sqrt{50}$
$d_3=\sqrt{(-2-10)^2+(4-8)^2}=\sqrt{144+16}=\sqrt{160}$
$d_1=d_2$ so the triangle is isosceles.

8. $a=\sqrt{(2-4)^2+(-4-0)^2}=\sqrt{4+16}=\sqrt{20}$
$b=\sqrt{(4-8)^2+(0+2)^2}=\sqrt{16+4}=\sqrt{20}$
$c=\sqrt{(2-8)^2+(-4+2)^2}=\sqrt{36+4}=\sqrt{40}$

$a^2+b^2=c^2$, so the triangle is a right triangle.

9. (–1, –1), (–1, 3); (7, –1), (7, 3); (1, 1), (5, 1)

10. $\sqrt{(x-3)^2+(0-1)^2}=\sqrt{(x-6)^2+(0-4)^2}$;

$$x^2-6x+10=x^2-12x+52$$
$$6x=42$$
$$x=7\Rightarrow(7,0)$$

11. $\left(\frac{-2+4}{2},\frac{-2+3}{2}\right)=\left(1,\frac{1}{2}\right)$;

$$d=\sqrt{(1+2)^2+\left(\frac{1}{2}-3\right)^2}=\sqrt{9+\frac{25}{4}}\approx 3.91$$

12. midpoint of $AB=\left(\frac{1+2}{2},\frac{3+6}{2}\right)=\left(\frac{3}{2},\frac{9}{2}\right)$

midpoint of $CD=\left(\frac{4+3}{2},\frac{7+4}{2}\right)=\left(\frac{7}{2},\frac{11}{2}\right)$

$$d=\sqrt{\left(\frac{3}{2}-\frac{7}{2}\right)^2+\left(\frac{9}{2}-\frac{11}{2}\right)^2}$$
$$=\sqrt{4+1}=\sqrt{5}\approx 2/24$$

13. $(x-1)^2+(y-1)^2=1$

14. $(x+2)^2+(y-3)^2=4^2$

$(x+2)^2+(y-3)^2=16$

15. $(x-2)^2+(y+1)^2=r^2$

$(5-2)^2+(3+1)^2=r^2$

$r^2=9+16=25$

$(x-2)^2+(y+1)^2=25$

16. $(x-4)^2+(y-3)^2=r^2$

$(6-4)^2+(2-3)^2=r^2$

$r^2=4+1=5$

$(x-4)^2+(y-3)^2=5$

17. center $=\left(\frac{1+3}{2},\frac{3+7}{2}\right)=(2,5)$

radius $=\frac{1}{2}\sqrt{(1-3)^2+(3-7)^2}=\frac{1}{2}\sqrt{4+16}$

$=\frac{1}{2}\sqrt{20}=\sqrt{5}$

$(x-2)^2+(y-5)^2=5$

18. Since the circle is tangent to the x-axis, $r=4$.

$(x-3)^2+(y-4)^2=16$

19. Substitute $x=\frac{1}{4}$ into the equation and solve for y.

$\left(-\frac{3}{4}\right)^2+(y-1)^2=1$

$(y-1)^2=\frac{7}{16}$

$y-1=\pm\frac{\sqrt{7}}{4}$

$y=1\pm\frac{\sqrt{7}}{4}$

20. Substitute $y=1$ into the equation and solve for x.

$(x-1)^2+(0)^2=1$

$x-1=\pm 1$

$x=0,2$

21. $x^2+2x+10+y^2-6y-10=0$

$$x^2+2x+y^2-6y=0$$
$$(x^2+2x+1)+(y^2-6y+9)=1+9$$
$$(x+1)^2+(y-3)^2=10$$

center $=(-1,3)$; radius $=\sqrt{10}$

22. $x^2+y^2-6y=36$

$$x^2+(y^2-6y+9)=16+9$$
$$x^2+(y-3)^2=25$$

center $=(0,3)$; radius $=\sqrt{5}$

23. $x^2+y^2-12x+35=0$

$$x^2-12x+y^2=-35$$
$$(x^2-12x+36)+y^2=-35+36$$
$$(x-6)^2+y^2=1$$

center $=(6,0)$; radius $=1$

24. $x^2+y^2-10x+10y=0$

$$(x^2-10x+25)+(y^2+10y+25)=25+25$$
$$(x-5)^2+(y+5)^2=50$$

center $=(5,-5)$; radius $=\sqrt{50}=5\sqrt{2}$

25. $4x^2+16x+15+4y^2+6y=0$

$$4(x^2+4x+4)+4\left(y^2+\frac{3}{2}y+\frac{9}{16}\right)=-15+16+\frac{9}{4}$$