CONSTRAINT THEORY

Multidimensional Mathematical Model Management

GEORGE FRIEDMAN

IFSR International Series on Systems Science and Engineering
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CONSTRAINT THEORY

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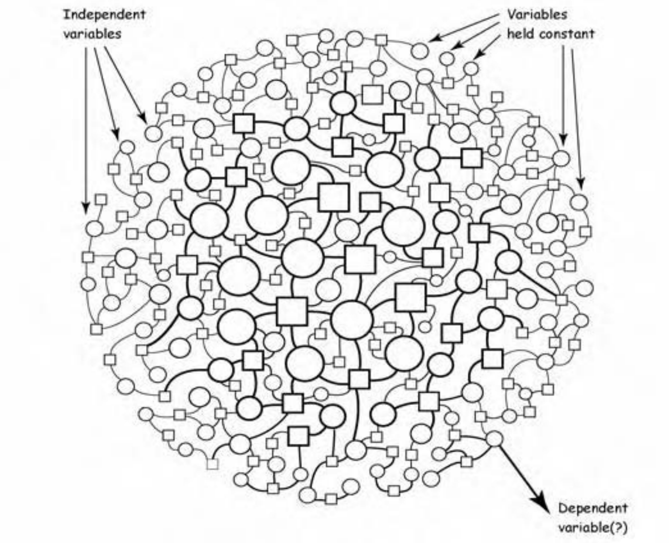
MULTIDIMENSIONAL MATHEMATICAL MODEL MANAGEMENT

George J. Friedman



Springer

Library of Congress Cataloging-in-Publication Data

Friedman, George J.
Constraint theory: multidimensional mathematical model management/by George J. Friedman.
p. cm. — (International series on systems science and engineering; v. 23)
Includes bibliographical references and index.
ISBN 0-387-23418-7
1. Systems engineering—Mathematical models. 2. Theory of constraints (Management)
I. Title. II. IFSR international series on systems science and engineering; v. 23.

TA168.F75 2005
620Rps.001′1—dc22
2004062569

ISBN 0-387-23418-7 (Hardbound) ISSN 1574-0463 Printed on acid-free paper.

Printed in the United States of America (SBA)

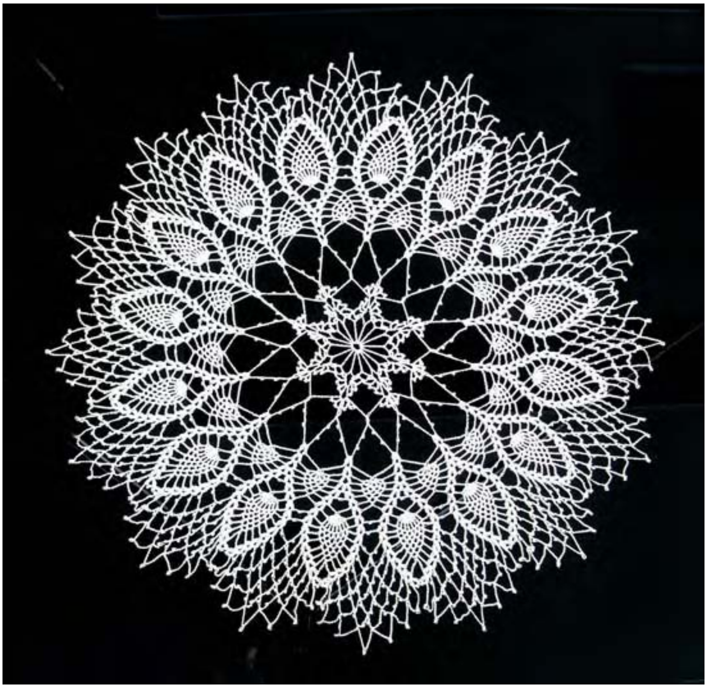
9 8 7 6 5 4 3 2 1

springeronline.com

To my wife Ruthanne,
who helped me find
the time and priority
for my mathematical dreams

Fronticepiece

24” Doily, designed and constructed by Regina Oberlander (the author’s mother) circa 1915 in Chinedeev, near Muncacevo, Austro-Hungarian Empire.



A single circuit cluster with over 1,000 independent simple circuits (thus there are 1000 more edges than vertices)

Preface

At first glance, this might appear to be a book on mathematics, but it is really intended for the practical engineer who wishes to gain greater control of the multidimensional mathematical models which are increasingly an important part of his environment. Another feature of the book is that it attempts to balance left- and right-brain perceptions; the author has noticed that many graph theory books are disturbingly light on actual topological pictures of their material.

One thing that this book is not is a depiction of the Theory of Constraints, as defined by Eliyahu Goldratt in the 1980's. Constraint Theory was originally defined by the author in his PhD dissertation in 1967 and subsequent papers written over the following decade. It strives to employ more of a mathematical foundation to complexity than the Theory of Constraints. This merely attempts to differentiate this book from Goldratt's work, not demean his efforts. After all, the main body of work in the field of Systems Engineering is still largely qualitative[1].

[1] The author has been frequently asked why there was such a dearth of constraint theory publication after the 1970's. The answer is that he was working on several classified aerospace programs wherein publication of any kind was most difficult. He is too humble to compare himself with the legendary John von Neumann, but is encouraged to observe that von Neumann's first book on Game Theory was only published decades after his first paper on the subject.

Acknowledgments

Over the past several years, Constraint Theory has been a substantial part of a special graduate course taught by the author in the Systems Architecting and Engineering program at the University of Southern California, where he is an adjunct professor. The feedback from this group of bright graduate students was invaluable. Special thanks are given to these graduate students who performed research studies – summarized in appendix A – on constraint theory: Leila Habibibabadi, Kathya Zamora-Diaz and Elliott Morgan [1].

Extra special thanks are given to the author's son, Gary L. Friedman who provided the extremely valuable service of intellectual reflector, editor and digital formal manager. Sincere appreciations also go out to Iduol Beny and Yin Yan Shum, the book's illustrators who worked tirelessly and have an exceptional eye for detail. Everlasting appreciation to Professor Cornelius Leondes, the author's Ph.D. committee chairman who guided his research and had the faith to support him several years longer than the system deemed proper. Finally, the author is deeply grateful to John Warfield for his decades of encouragement and insightful advice.

Introduction

Many thousands of papers have been written about the accelerating pace of increased complexity and interactivity in virtually every walk of life in the developed world. Domains which previously could have been studied and managed separately – such as energy, the environment and economics -- must now be dealt with as intimately intertwined disciplines. With its multitude of additional capabilities, complex systems also provide a treacherous array of fragile failure modes, and both the development and operation of new systems are an increasing challenge to the systems engineer. Advanced technology is the primary driving force behind the increasing complexity and the enthusiastic pushing of immature technologies is behind most of the early failures in the development phases.

Perhaps the most significant advanced technology employed in new complex systems is the computer science family with its ancillary disciplines of communications and software. Fortunately, computer science also represents a major opportunity to control the design and operation of complex systems because of its ability to perform multi-dimensional modeling to any level of detail desired. Math models have been used in support of every phase of systems engineering, including requirements management, preliminary design, interface and integration, validation and test, risk management, manufacturing, reliability and maintainability, training, configuration management and developing virtual universes to measure customer preferences prior to the implementation of the design. Properly used, the enormous power of modern computers can even furnish researchers with a synthetic world where theories can be tried and tested on validated models, thus avoiding far more expensive tests in the real world. A wide variety of questions – or “tradeoffs’ – can be asked of the models and, at least in theory, the analyst has a free choice as to which computations he wishes to observe and which variables he desires to be independent. Philosophically, it can even be argued that the math model employed in this fashion provides the technologist a virtual extension of the scientific method itself.

Those who have actually wrestled with large-scale models will complain that the above description is far too rosy. Submodels which are developed by separate organizations are normally very difficult to integrate into an overall working model; they often must be dealt with as "islands of automation." The greatest of care must be taken to make sure that the definition of each variable is clear and agreeable to every member of the team. In general, it is difficult to distinguish between a model and the computer program, and if a computational request is made which reverses dependent and independent variables, then the model must be reprogrammed. To say the least, much diligent effort must be undertaken to obtain the many advantages promised by mathematical modeling.

However, even after the diligence, there exists a much deeper problem that often diminishes the utility of math modeling; it is associated with the traditional "well posed" problem in mathematics. We need to know whether the model is internally consistent and whether computational requests made on it are allowable. The alarming facts are that models constructed by diverse teams – and this is normally the case for very large models – have internal inconsistencies and that most of the possible computational requests which can be made on even consistent models are not allowable. This problem is the domain addressed by Constraint Theory and is the subject of this book.

Chapter One provides an example of low dimension, showing how problems of consistency and computational allowability can arise in even simple situations. The reader is introduced to the two main characters of the book – an experienced manager and an analyst – whose dialogue will hopefully illuminate the book's many concepts. The bipartite graph is introduced, as are a few simple rules. However, the analyst argues that, in order to expand the tools to models of very high dimension, and in order to trust the reliability of these tools, the theory must be based on a more rigorous foundation. "Only the simplest 5% of graph theory and set theory are required", he claims.

Chapter Two begins to establish the rigorous foundation by defining four "views" of a mathematical model: 1) set theoretic, 2) submodel family, 3) bipartite graph, and 4) constraint matrix. The first two views are full models; the last two views are metamodels. Then, rigorous definitions of consistency and computational allowability are made in the context of these views.

Chapter Three discusses the similarities between language and mathematics and provides some general consistency and computability results with respect to any class of relation. In order to provide a basis for the next three chapters, three classes of exhaustive and mutually exclusive relations are defined: discrete, continuum, and interval.

Chapter Four addresses the constraint theoretic properties of *regular* relations, the most important type within the continuum class, and the most often employed in the development of multidimensional math models. The simple rules presented in Chapter 1 are rigorously proved employing the foundations of Chapters 2 and 3. The topological properties of the bipartite graph are analyzed to provide key conclusions of the model's consistency and computational properties.

A specific type of subgraph within the bipartite graph, called the *Basic Nodal Square (BNS)* is identified as the "kernel of intrinsic constraint" and is accused of being the culprit in model inconsistency and unallowable computability. Trivially easy computations on the bipartite graph – such as circuit rank and constraint potential – are shown to have enormous utility in locating the BNSs which hide in tangled circuit clusters. A constraint theory toolkit is provided to employ the rules and theorems in an orderly manner and which can find BNSs trillions of times faster than brute force approaches. This chapter is the longest in the book and represents the core of constraint theory at its present stage.

Chapter Five addresses the constraint properties of *discrete* and *interval* functions such as those from Boolean algebra, logic and inequalities. These classes of relations are less important in support of modern math modeling, but strangely, it was the first that the author studied in his development of Constraint Theory. It was easier for him to imagine multidimensional sets of points than multidimensional sets of continuous functions. Interval relations require the greatest interaction between models and metamodels, and the concept of constraint potential is less useful than for regular relations.

Chapter Six provides a compact structure of constraint theory. All postulates, definitions and theorems are listed and their logical interrelationships are displayed in the form of quasi-bipartite graphs.

Chapter Seven presents detailed examples of the application of constraint theory to the areas of operations analysis, kinematics of free-fall weapon delivery systems and the dynamics of deflecting asteroids with mass drivers.

Chapter Eight summarizes the book and provides the manager and analyst a final opportunity to dialogue and discuss their common background.

Problems for the interested student are presented at the end of most chapters, so this book could be employed as a text for a graduate course -- or senior level undergraduate course -- in Systems Engineering or mathematical modeling.

Of course, a complete list of *references* is provided, as well as an *index.*

Several *appendices* treat detailed material to a depth that would slow down the natural rhythm of the exposition if they were included in the chapters themselves. *Appendix A* is noteworthy in that it summarizes the research projects on "computational request disappointments." On models approximately the size of Chapter 1's "simple example" – eight variables – the percentage of allowable computational requests based on the total number of possible computational requests is only on the order of 10%. It is presently "Friedman's conjecture" that as the dimensionality, K, of the model increases, the number of allowable computational requests also increases, perhaps as fast as the square of the model's dimension or K^2. However, the number of possible computational requests increases far faster: 2^K. Thus, for a 100-dimension model, only 10^{-26} of all possible computational requests will be allowable! Models of thousands of dimensions have been built and are planned; so the ratio of allowable to possible computational requests is enormously worse that even this incredibly low number. The technologist who wishes to gain maximum benefit from asking his model to perform any computation his imagination conjures up will certainly be disappointed! A tool such as constraint theory which will lead him to the 10,000 computational requests (K=100) or 1,000,000 requests (K=1,000) which are allowable should be valuable.

Appendix B provides a very brief overview of graph theory with the objective of justifying why the bipartite graph was chosen as the primary metamodel for constraint theory.

Appendix C describes the rigorous logic of the difference between "if and only if" and "if" types of theorems. Most of constraint theory's theorems are of the latter category – a source of confusion to many students.

Appendix D develops the similarity between vector spaces and graph theory circuits. The concept of independence in both worlds is strikingly similar and the ability to analyze circuits in graph theory has powerful ramifications in constraint theory because basic nodal squares – the source in intrinsic constraint – are always found in circuit clusters.

A Warmup Problem in Complexity

This book makes substantial use of a mathematical structure from graph theory called a bipartite graph. In the past, bipartite graphs have been employed to solve "pairing" problems associated with various social situations such as picnics or dinner parties.

Out of respect for this tradition, let us consider a set of five men – named Jack, Jake, Jude, Juan, and Jobe – and a set of five women – named Jane, Joan, June, Jean, and Jenn. Let us define a *relationship pattern* as a complete description of all heterosexual relationships between the five men and five women. For example:

- In the *communal* pattern, every man has a relationship with every woman. There is one such pattern.
- In the *celibacy* pattern, none of the men have a relationship with any of the women. Again, there is one such pattern.
- In the *male harem* patterns, one of the men has a relationship with each of the women, but all the other men are devoid of relationships, except perhaps to be eunuchs. There are five such patterns. Similarly, there are five possible *female harem* patterns.
- In the *monogamy* patterns, each man has a relationship with exactly one woman and vice versa. There are 5!=120 such distinct patterns.

And so on. There are many more patterns. The question is: *What is the total number of possible heterosexual relationship patterns between five men and five women?*

The answer – discussed in Chapter 4 and Appendix A – may surprise you: it's over 30 million (!). It certainly surprised the author and changed an important objective of his research agenda. Moreover it represents the hidden depths possible in apparently simple problems of low dimension as well as a challenge to one's belief in intuition or rational mathematics.

About the Author

George Friedman is an Adjunct Professor in the Industrial and Systems Engineering Department of the School of Engineering of the University of Southern California. He has developed and taught graduate courses in systems engineering with emphasis on the management of complexity and decision science. This book is the product of one of these courses.

He has had over 45 years of experience in industry, retiring from the Northrop Corporation as their Corporate Vice President of Engineering and Technology. He worked on a wide variety of aerospace programs and served as a consultant to all branches of the Department of Defense, NASA, the National Science Foundation, Department of Energy as well as to the NATO industrial advisory group.

He was a founder of the International Council on Systems Engineering (INCOSE), served as its third president, was elected a fellow and is on the editorial board of INCOSE's journal, *Systems Engineering.*

He has also been a member of the Institute of Electrical and Electronic Engineers (IEEE) since its formation from the IRE and AIEE, was elected a fellow and was the vice president for publications of the *IEEE Transactions on Aerospace and Electronics Systems.* He received the Baker Prize for the best paper published by all societies of the IEEE in 1970 – the subject of the paper was Constraint Theory.

He is presently the director of research at the Space Studies Institute at Princeton, and has supported several new technologies associated with the long range development of space.

He received the Bachelor's degree in engineering at the University of California at Berkeley and the Masters and Doctorate at UCLA. The topic of his PhD dissertation was constraint theory [2, 3].

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Chapter 1

MOTIVATIONS

What is Constraint Theory and why is it important?

1.1 TRENDS AND PROBLEMS IN SYSTEM TECHNOLOGIES

Gone forever are the simple days! Virtually every identifiable trend is driving humanity's enterprises into more intimate interaction and conflict. Increased population, accelerated exploitation of resources, and expanded transportation have brought the previously decoupled worlds of economics, energy and the environment into direct conflict. With the greater efficiency of travel and communication, the emergence of global marketplaces and the revolution in military strategies, the international world is incredibly more interactive, multidimensional and complex than even a decade ago. Locally, we observe ever tighter coupling between emerging problems in crime, poverty, education, health and drug misuse. All these issues have been aggravated by an explosion of new technology and -- especially in the United States -- a compulsion to force these new technologies into early and often simultaneous application. The most vigorous of these advancing technologies -- digital computation -- brings with it an unexpected complexity challenge: software and the management of highly complex and multidimensional mathematical models.

Fortunately, this most rapidly advancing technology of computer science not only adds to the complexity of *designed* systems, it also contributes enormously to *designing* these systems themselves. A host of new "computer assisted" software packages are published each year, running the gamut from Computer Assisted Design (CAD), Computer Assisted Engineering (CAE), Computer Assisted Systems Engineering (CASE), Computer Assisted Manufacturing (CAM), Computer Assisted Instruction