CliffsQuickReview™ Calculus

Anton/Bivens/Davis Version

By Bernard V. Zandy, MA and
Jonathan J. White, MS

WILEY
Wiley Publishing, Inc.

About the Authors

Bernard V. Zandy, MA, Professor of Mathematics at Fullerton College in California has been teaching secondary and college level mathematics for 34 years. A co-author of the Cliffs PSAT and SAT Preparation Guides, Mr. Zandy has been a lecturer and consultant for Bobrow Test Preparation Services, conducting workshops at California State University and Colleges since 1977.

Jonathan J. White has a BA in mathematics from Coe College and an MS in mathematics from the University of Iowa. He is currently pursuing a PhD in Mathematics Pedagogy and Curriculum Research at the University of Oklahoma.

Publisher's Acknowledgments

Editorial

Project Editor: Brian Kramer

Acquisitions Editor: Sherry Gomoll

Technical Editor: Dale Johnson

Composition

Indexer: TECHBOOKS Production Services

Proofreader: Joel K. Draper

Wiley Publishing Composition Services

CliffsQuickReview™ Calculus

Published by

Wiley Publishing, Inc.

909 Third Avenue

New York, NY 10022

www.wiley.com

www.cliffsnotes.com

Library of Congress Control Number available from the Library of Congress

ISBN: 0-7645-4225-7

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

1O/QS/QW/QT/IN

Published by Wiley Publishing, Inc., New York, NY

Published simultaneously in Canada

Table of Contents

Introduction **1**
Why You Need This Book 1
How to Use This Book 1
Using Calculus 7e by Anton/Bivens/Davis (ABD) with CQR 2

Chapter 1: Review Topics **3**
Interval Notation 3
Absolute Value 4
Functions 4
Linear Equations 5
Trigonometric Functions 8

Chapter 2: Limits **14**
Intuitive Definition 14
Evaluating Limits 16
One-sided Limits 18
Infinite Limits 19
Limits at Infinity 21
Limits Involving Trigonometric Functions 23
Continuity 24

Chapter 3: The Derivative **29**
Definition 29
Differentiation Rules 32
Trigonometric Function Differentiation 34
Chain Rule 35
Implicit Differentiation 37
Higher Order Derivatives 39
Differentiation of Inverse Trigonometric Functions 40
Differentiation of Exponential and Logarithmic Functions 41

Chapter 4: Applications of the Derivative **43**
Tangent and Normal Lines 43
Critical Points 44
Extreme Value Theorem 45
Mean Value Theorem 46
Increasing/Decreasing Functions 48
First Derivative Test for Relative Extrema 49
Second Derivative Test for Relative Extrema 50
Concavity and Points of Inflection 51
Maximum/Minimum Problems 52
Distance, Velocity, and Acceleration 55
Related Rates of Change 56
Differentials 58

Chapter 5: Integration 63
Antiderivatives/Indefinite Integrals 63
Integration Techniques 64
Basic formulas 64
Substitution and change of variables 66
Integration by parts 68
Trigonometric integrals 69
Distance, Velocity, and Acceleration 73
Definite Integrals 75
Definition of definite integrals 75
Properties of definite integrals 78
The Fundamental Theorem of Calculus 80
Definite integral evaluation 82

Chapter 6: Applications of the Definite Integral 88
Area 88
Volumes of Solids with Known Cross Sections 93
Volumes of Solids of Revolution 96
Disk method 96
Washer method 97
Cylindrical shell method 99
Arc Length 101

CQR Review 104

CQR Resource Center 109

Glossary 113

Appendix: Using Graphing Calculators in Calculus 116
Limits 116
Derivatives 117
Integrals 119

Index 121

CORRELATION GUIDE: *CLIFFSQUICKREVIEW CALCULUS* WITH *CALCULUS 7E* BY ANTON, BIVENS, DAVIS

CliffsQuickReview Calculus		***Calculus 7E***
Chapter	***Topic Title***	
1	Interval Notation	Appendix A
1	Absolute Value	Appendix B
1	Functions	Sections 1.1, 1.2
1	Linear Equations	Appendix C
1	Trigonometric Functions	Appendix E
2	Intuitive Definition	Section 2.1
2	Evaluating Limits	Section 2.2
2	One-Sided Limits	Sections 2.1, 2.2
2	Infinite Limits	Sections 2.1, 2.2
2	Limits at Infinity	Sections 2.1, 2.3
2	Limits Involving Trigonometric Functions	Section 2.6
2	Continuity	Section 2.5
3	Definition	Section 3.2
3	Differentiation Rules	Section 3.3
3	Trigonometric Function Differentiation	Section 3.4
3	Chain Rule	Section 3.5
3	Implicit Differentiation	Section 3.6
3	Higher Order Derivatives	Section 3.3
3	Differentiation of Inverse Trigonometric Functions	Section 4.4
3	Differentiation of Exponential and Logarithmic Functions	Section 4.3

(continued)

CliffsQuickReview Calculus		***Calculus 7E***
Chapter	***Topic Title***	
4	Tangent and Normal Lines	Section 3.2
4	Critical Points	Section 5.2
4	Extreme Value Theorem	Section 5.5
4	Mean Value Theorem	Section 5.8
4	Increasing/Decreasing Functions	Section 5.1
4	First Derivative Test for Local Extrema	Section 5.2
4	Second Derivative Test for Local Extrema	Section 5.2
4	Concavity and Points of Inflection	Section 5.1
4	Maximum/Minimum Problems	Section 5.6
4	Distance, Velocity, Acceleration	Section 5.4
4	Related Rates of Change	Section 3.7
4	Differentials	Section 3.8
5	Antiderivatives/Indefinite Integrals	Section 6.2
5	Integration Techniques	Section 6.2
5	Substitution and Change of Variables	Section 6.3
5	Integration by Parts	Section 8.2
5	Trigonometric Integrals	Section 8.3
5	Trigonometric Substitutions	Section 8.4
5	Distance, Velocity, and Acceleration	Section 6.7
5	Definition of Definite Integrals	Section 6.5
5	Properties of Definite Integrals	Sections 6.5, 6.6
5	The Fundamental Theorem of Calculus	Section 6.6
5	Definite Integral Evaluation	Section 6.8

Chapter	Topic Title	
6	Area	Section 7.1
6	Volumes of Solids with Known Cross Sections	Section 7.2
6	Disk Method	Section 7.2
6	Washer Method	Section 7.2
6	Cylindrical Shell Method	Section 7.3
6	Arc Length	Section 7.4

CORRELATION GUIDE: *CLIFFSQUICKREVIEW CALCULUS* WITH *CALCULUS: EARLY TRANSCENDENTALS 7E* BY ANTON, BIVENS, DAVIS

CliffsQuickReview Calculus		***Calculus 7E***
Chapter	**Topic Title**	
1	Interval Notation	Appendix A
1	Absolute Value	Appendix B
1	Functions	Sections 1.1, 1.2
1	Linear Equations	Appendix C
1	Trigonometric Functions	Appendix E
2	Intuitive Definition	Section 2.1
2	Evaluating Limits	Section 2.2
2	One-Sided Limits	Sections 2.1, 2.2
2	Infinite Limits	Sections 2.1, 2.2
2	Limits at Infinity	Sections 2.1, 2.3

(continued)

CliffsQuickReview Calculus		***Calculus 7E***
Chapter	***Topic Title***	
2	Limits Involving Trigonometric Functions	Section 2.6
2	Continuity	Section 2.5
3	Definition	Section 3.2
3	Differentiation Rules	Section 3.3
3	Trigonometric Function Differentiation	Section 3.4
3	Chain Rule	Section 3.5
3	Implicit Differentiation	Section 3.6
3	Higher Order Derivatives	Section 3.3
3	Differentiation of Inverse Trigonometric Functions	Section 4.4
3	Differentiation of Exponential and Logarithmic Functions	Section 4.3
4	Tangent and Normal Lines	Section 3.2
4	Critical Points	Section 5.2
4	Extreme Value Theorem	Section 5.5
4	Mean Value Theorem	Section 5.8
4	Increasing/Decreasing Functions	Section 5.1
4	First Derivative Test for Local Extrema	Section 5.2
4	Second Derivative Test for Local Extrema	Section 5.2
4	Concavity and Points of Inflection	Section 5.1
4	Maximum/Minimum Problems	Section 5.6
4	Distance, Velocity, Acceleration	Section 5.4
4	Related Rates of Change	Section 3.7
4	Differentials	Section 3.8

Chapter	*Topic Title*	
5	Antiderivatives/Indefinite Integrals	Section 6.2
5	Integration Techniques	Section 6.2
5	Substitution and Change of Variables	Section 6.3
5	Integration by Parts	Section 8.2
5	Trigonometric Integrals	Section 8.3
5	Trigonometric Substitutions	Section 8.4
5	Distance, Velocity, and Acceleration	Section 6.7
5	Definition of Definite Integrals	Section 6.5
5	Properties of Definite Integrals	Sections 6.5, 6.6
5	The Fundamental Theorem of Calculus	Section 6.6
5	Definite Integral Evaluation	Section 6.8
6	Area	Section 7.1
6	Volumes of Solids with Known Cross Sections	Section 7.2
6	Disk Method	Section 7.2
6	Washer Method	Section 7.2
6	Cylindrical Shell Method	Section 7.3
6	Arc Length	Section 7.4

Introduction

Calculus is the mathematics of change. Any situation that involves quantities that change over time can be understood with the tools of calculus. **Differential calculus** deals with rates of change or slopes, and is explored in Chapters 3 and 4 of this book. **Integral calculus** handles total changes or areas, and is addressed in Chapters 5 and 6. Although it is not always immediately obvious, this mathematical notion of change is essential to many areas of knowledge, particularly disciplines like physics, chemistry, biology, and economics.

The prerequisites for learning calculus include much of high school algebra and trigonometry, as well as some essentials of geometry. If the formulas on the front side of the Pocket Guide (the cardstock page right inside the front cover) and topics covered in Chapter 1 are familiar to you, then you probably have sufficient background to begin learning calculus. If some of those are unfamiliar, or just rusty for you, then *CliffsQuickReview Geometry, CliffsQuickReview Algebra,* or *CliffsQuickReview Trigonometry* may be valuable starting points for you.

Why You Need This Book

Can you answer yes to any of these questions?

- Do you need to review the fundamentals of calculus fast?
- Do you need a course supplement to calculus?
- Do you need a concise, comprehensive reference for calculus?

If so, then *CliffsQuickReview Calculus* is for you!

How to Use This Book

You can use this book in any way that fits your personal style for study and review—you decide what works best with your needs. You can either read the book from cover to cover or just look for the information you want and put it back on the shelf for later. Here are just a few ways you can use this book:

- Read the book as a stand-alone textbook to learn all the major concepts of calculus.

- Use the Pocket Guide to find often-used formulas, from calculus and other relevant formulas from algebra, geometry and trigonometry.
- Refer to a single topic in this book for a concise and understandable explanation of an important idea.
- Get a glimpse of what you'll gain from a chapter by reading through the "Chapter Check-In" at the beginning of each chapter.
- Use the Chapter Checkout at the end of each chapter to gauge your grasp of the important information you need to know.
- Test your knowledge more completely in the CQR Review and look for additional sources of information in the CQR Resource Center.
- Review the most important concepts of an area of calculus for an exam.
- Brush up on key points as preparation for more advanced mathematics.

Using Calculus 7e by Anton/Bivens/Davis (ABD) with CQR

- ABD to CQR. If you are reading the ABD text, use the Correlation Guide in the front of the CQR (right before page 1) to quickly find the corresponding topic in your CliffsQuickReview.
- CQR to ABD. If you are using your CQR, you can easily find additional explanation or examples in Anton/Bivens/Davis—section heads within this CilffsQuickReview are followed by an icon indicating on what page you can find more help in Anton/Bivens/Davis.

Being a valuable reference source also means it's easy to find the information you need. Here are a few ways you can search for topics in this book:

- Look for areas of interest in the book's Table of Contents, or use the index to find specific topics.
- Use the glossary to find key terms fast. This book defines new terms and concepts where they first appear in the chapter. If a word is boldfaced, you can find a more complete definition in the book's glossary.
- Flip through the book looking for subject areas at the top of each page.
- Or browse through the book until you find what you're looking for—we organized this book to gradually build on key concepts.

Chapter 1
REVIEW TOPICS

Chapter Check-In

- ❑ Reviewing functions
- ❑ Using equations of lines
- ❑ Reviewing trigonometric functions

Certain topics in algebra, geometry, analytical geometry, and trigonometry are essential in preparing to study calculus. Some of them are briefly reviewed in the following sections.

Interval Notation

The set of real numbers (R) is the one that you will be most generally concerned with as you study calculus. This set is defined as the union of the set of rational numbers with the set of irrational numbers. Interval notation provides a convenient abbreviated notation for expressing intervals of real numbers without using inequality symbols or set-builder notation.

The following lists some common intervals of real numbers and their equivalent expressions, using set-builder notation:

$$(a, b) = \{x \in R: a < x < b\}$$

$$[a, b] = \{x \in R: a \leq x \leq b\}$$

$$[a, b) = \{x \in R: a \leq x < b\}$$

$$(a, b] = \{x \in R: a < x \leq b\}$$

$$(a, +\infty) = \{x \in R: x > a\}$$

$$[a, +\infty) = \{x \in R: x \geq a\}$$

$$(-\infty, b) = \{x \in R: x < b\}$$

$$(-\infty, b] = \{x \in R: x \leq b\}$$

$$(-\infty, +\infty) = \{x \in R\}$$

Note that an infinite end point $(\pm\infty)$ is never expressed with a bracket in interval notation because neither $+\infty$ nor $-\infty$ represents a real number value.

Absolute Value

The concept of absolute value has many applications in the study of calculus. The absolute value of a number *a*, written $|a|$ may be defined in a variety of ways. On a real number line, the absolute value of a number is the distance, disregarding direction, that the number is from zero. This definition establishes the fact that the absolute value of a number must always be nonnegative—that is, $|a| \geq 0$.

A common algebraic definition of absolute value is often stated in three parts, as follows:

$$|a| = \begin{cases} a, a > 0 \\ 0, a = 0 \\ -a, a < 0 \end{cases}$$

Another definition that is sometimes applied to calculus problems is

$$|a| = \sqrt{a^2}$$

or the principal square root of a^2. Each of these definitions also implies that the absolute value of a number must be a nonnegative.

Functions

A **function** is defined as a set of ordered pairs (x,y), such that for each first element *x*, there corresponds one and only one second element *y*. The set of first elements is called the *domain* of the function, while the set of second elements is called the *range* of the function. The domain variable is referred to as the independent variable, and the range variable is referred to as the dependent variable. The notation $f(x)$ is often used in place of *y* to indicate the value of the function *f* for a specific replacement for *x* and is read "*f* of *x*" or "*f* at *x*."

Geometrically, the graph of a set or ordered pairs (x,y) represents a function if any vertical line intersects the graph in, at most, one point. If a vertical line were to intersect the graph at two or more points, the set would have one x value corresponding to two or more y values, which clearly contradicts the definition of a function. Many of the key concepts and theorems of calculus are directly related to functions.

Example 1-1: The following are some examples of equations that define functions.

(a) $y = f(x) = 3x + 1$
(b) $y = f(x) = x^2$
(c) $y = f(x) = |x| - 5$
(d) $y = f(x) = -3$
(e) $y = f(x) = \frac{x-3}{x^2+4}$
(f) $y = f(x) = \sqrt[3]{2x+9}$
(g) $y = f(x) = \frac{6}{x}$
(h) $y = \tan x$
(i) $y = \cos 2x$

Example 1-2: The following are some equations that do not define functions; each has an example to illustrate why it does not define a function.

(a) $x = y^2$; If $x = 4$, then $y = 2$ or $y = -2$
(b) $x = |y + 3|$; If $x = 2$, then $y = -5$ or $y = -1$
(c) $x = -5$; If $x = -5$, then y can be any real number.
(d) $x^2 + y^2 = 25$; If $x = 0$, then $y = 5$ or $y = -5$.
(e) $y = \pm\sqrt{x+4}$; If $x = 5$, then $y = +3$ or $y = -3$.
(f) $x^2 - y^2 = 9$; If $x = -5$, then $y = 4$ or $y = -4$.

Linear Equations App C

A **linear equation** is any equation that can be expressed in the form $Ax + By + C = 0$, where A and B are not both zero. Although a linear equation may not be expressed in this form initially, it can be manipulated algebraically to this form. The slope of a line indicates whether the line slants up or down to the right or is horizontal. The slope is usually denoted by the letter m and is defined in a number of ways:

$$\begin{aligned} m &= \frac{\text{rise}}{\text{sun}} \\ &= \frac{\text{vertical change}}{\text{horizontal change}} \\ &= \frac{y \text{ value change}}{x \text{ value change}} \\ &= \frac{\Delta y}{\Delta x} \\ &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{y_1 - y_2}{x_1 - x_2} \end{aligned}$$

Note that for a vertical line, the x value would remain constant, and the horizontal change would be zero; hence, a vertical line is said to have no slope or its slope is said to be **nonexistent** or **undefined.** All nonvertical lines have a numerical slope with a positive slope indicating a line slanting up to the right, a negative slope indicating a line slanting down to the right, and a slope of zero indicating a horizontal line.

Example 1-3: Find the slope of the line passing through (–5, 4) and (–1, –3).

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{(-3) - (4)}{(-1) - (-5)} \\ &= -\frac{7}{4} \end{aligned}$$

The line, then, has a slope of –7/4.

Some forms of expressing linear equations are given special names that identify how the equations are written. Note that even though these forms appear to be different from one another, they can be algebraically manipulated to show they are equivalent.

Any nonvertical lines are parallel if they have the same slopes, and conversely lines with equal slopes are parallel. If the slopes of two lines L_1 and L_2 are m_1 and m_2, respectively, then L_1 is parallel to L_2 if and only if $m_1 = m_2$.

Two nonvertical, nonhorizontal lines are perpendicular if the product of their slopes is –1, and conversely, if the product of their slopes is –1, the lines are perpendicular. If the slopes of two lines L_1 and L_2 are m_1 and m_2, respectively, then L_1 is perpendicular to L_2 if and only if $m_1 \cdot m_2 = -1$.

You should note that any two vertical lines are parallel and a vertical line and a horizontal line are always perpendicular.

The **general** or **standard form** of a linear equation is $Ax + By + C = 0$, where A and B are not both zero. If $B = 0$, the equation takes the form x = constant and represents a vertical line. If $A = 0$, the equation takes the form y = constant and represents a horizontal line.

Example 1-4: The following are some examples of linear equations expressed in general form:

(a) $2x + 5y - 10 = 0$

(b) $x - 4y = 0$

(c) $x + 3 = 0$

(d) $y - 6 = 0$

The **point-slope form** of a linear equation is $y - y_1 = m(x - x_1)$ when the line passes through the point (x_1, y_1) and has a slope m.

Example 1-5: Find an equation of the line through the point (3,4) with slope –2/3.

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{2}{3}(x - 3)$$

$$y - 4 = -\frac{2}{3}x + 2$$

$$y = -\frac{2}{3}x + 6$$

$$3y = -2x + 18$$

$$2x + 3y - 18 = 0 \quad \text{(general form)}$$

The **slope-intercept form** of a linear equation is $y = mx + b$ when the line has y-intercept $(0,b)$ and slope m.

Example 1-6: Find an equation of the line that has a slope 4/3 and crosses y-axis at –5.

$$y = mx + b$$

$$y = \frac{4}{3}x + (-5)$$

$$3y = 4x - 15$$

$$4x - 3y - 15 = 0 \quad \text{(general form)}$$

The **double intercept form** of a linear equation is $x/a + y/b = 1$ when the line has x-intercept $(a,0)$ and y-intercept $(0,b)$.

Example 1-7: Find the double intercept and slope intercept form of an equation of the line that crosses the x-axis at –2 and the y-axis at 3.

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{-2} + \frac{y}{3} = 1 \text{ (double intercept form)}$$

$$y = \frac{3}{2}y + 3 \text{ (slope intercept form)}$$

Trigonometric Functions

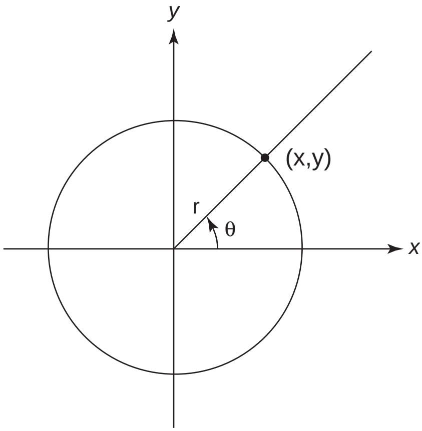
In trigonometry, angle measure is expressed in one of two units: degrees or radians. The relationship between these measures may be expressed as follows: $180^\circ = \pi$ radians.

To change degrees to radians, the equivalent relationship $1^\circ = \pi/180$ radians is used, and the given number of degrees is multiplied by $\pi/180$ to convert to radian measure. Similarly, the equation 1 radian $= 180^\circ/\pi$ is used to change radians to degrees by multiplying the given radian measure by $180/\pi$ to obtain the degree measure.

The six basic trigonometric functions may be defined using a circle with equation $x^2 + y^2 = r^2$ and the angle θ in standard position with its vertex at the center of the circle and its initial side along the positive portion of the x-axis (see Figure 1-1).

The trigonometric functions sine, cosine, tangent, cotangent, secant, and cosecant are defined as follows:

Figure 1-1 Defining the trigonometric functions.



$$\sin\theta = \frac{y}{r} = \frac{y}{\sqrt{x^2+y^2}}$$

$$\cos\theta = \frac{x}{r} = \frac{x}{\sqrt{x^2+y^2}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y}{x}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{x}{y}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{r}{x} = \frac{\sqrt{x^2+y^2}}{x}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{r}{y} = \frac{\sqrt{x^2+y^2}}{y}$$

It is essential that you be familiar with the values of these functions at multiples of 30°, 45°, 60°, 90°, and 180° (or in radians, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$, and π (See Table 1-1.) You should also be familiar with the graphs of the six trigonometric functions. Some of the more common trigonometric identities that are used in the study of calculus are as follows:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

$$\sin(\theta + 2\pi) = \sin\theta$$

$$\cos(\theta + 2\pi) = \cos\theta$$

$$\tan(\theta + \pi) = \tan\theta$$

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\sin(A - B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\cos(A - B) = \cos A\cos B + \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\begin{aligned}\cos 2\theta &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta\end{aligned}$$

$$\sin^2\frac{1}{2}\theta = \frac{1 - \cos\theta}{2}$$

$$\cos^2\frac{1}{2}\theta = \frac{1 + \cos\theta}{2}$$

The relationship between the angles and sides of a triangle may be expressed using the **Law of Sines** or the **Law of Cosines** (see Figure 1-2).