Collective Phenomena in Synchrotron Radiation Sources

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S. Khan

Collective Phenomena in Synchrotron Radiation Sources

Prediction, Diagnostics, Countermeasures

With 92 Figures

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Professor Dr. Shaukat Khan
Universität Hamburg
Institut für Experimentalphysik
Luruper Chaussee 149
22761 Hamburg, Germany

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Preface

This book is intended to dispel the notion that collective phenomena are an obscure and inaccessible topic. Having become increasingly important in modern storage rings, collective effects deserve to become part of the general education in accelerator physics.

I tried to write the book I was looking for when I started to study collective instabilities: an introduction that can be read from cover to cover with moderate effort and familiarizes the reader with the basic concepts. Self-contained appendices contain what I considered worth discussing but might impair the readability of the main text, such as a reminder of basic relations and some background information on selected topics. Finally, an extensive list of references was compiled (with some emphasis on accessibility of the source) in order to facilitate the search for more detailed information.

A previous version of the present text was submitted in German language as *Habilitationsschrift* – a postdoctoral thesis, which is a peculiarity of the German academic system – to the Humboldt University of Berlin. It grew from my occupation with collective phenomena over ten years at the third-generation synchrotron light source BESSY II. This has some consequences regarding the character of this text:

- There is a clear emphasis on synchrotron light sources and electron storage rings. However, many concepts (wake field, impedance, beam spectrum, beam lifetime, etc.) are quite general.
- The selection of material reflects my personal experience and some readers may want to skip over details that I considered important, while others may find that their favorite topic was not covered in sufficient detail.
- As suggested by the subtitle of the book, its content follows the logical sequence of prediction (in the design phase of a new facility), diagnostics (during commissioning, initial operation, and accelerator physics studies) and countermeasures (required for optimum routine operation).
- Most data and examples are taken from BESSY II, which makes the treatment monolithic and avoids confusion with varying machine parameters. In addition, references are given to direct the reader's attention to the work done and results obtained at other laboratories.

Synchrotron radiation sources *are* collective phenomena, since they are created by the coherent effort of many people. Therefore, I would like to thank all my colleagues who contributed to the successful operation of BESSY II. Many of them were directly involved in the work described in this book. In constructing and commissioning the multi-bunch feedback systems, I am particularly indepted to T. Knuth, who as a doctorate student did most of the work. Hardware and software contributions came from W. Anders, K. Bürkmann, V. Dürr, F. Falkenstern, R. Lange, J. Rahn, G. Schindhelm and others. I would also like to thank D. Krämer and P. Kuske for their continuous support and many helpful discussions. Last, but not least, I am grateful to the BESSY management, particularly to Prof. E. Jaeschke, for providing excellent working conditions with a considerable amount of freedom which made writing this book possible. All work at BESSY was funded by the Bundesministerium für Bildung und Forschung and by the Land Berlin.

In the context of the feedback projects, I would like to thank J. Fox, S. Prabhakar, D. Teytelman, A. Young (SLAC, Stanford, USA), W. Barry, J. Byrd, J. Corlett, G. Stover (LBNL, Berkeley, USA), A. Drago, A. Gallo, A. Ghigo, F. Marcellini and M. Serio (INFN-LNF, Frascati, Italy) for their friendly support and hospitality. Other work mentioned in this book profited from the expertise of K. Balewski (DESY, Hamburg, Germany), L. Palumbo (Università di Roma “La Sapienza”, Italy), H. Henke and T. Scholz (TU Berlin, Germany), T. Weiland (TU Darmstadt, Germany), and M. Zobov (INFN-LNF, Frascati, Italy).

It is a pleasure to thank C. Ascheron, physics editor at Springer, for his encouragement and patience. Finally, I have to apologize to my family, Doris and Theresa, for all the spoilt weekends.

Hamburg, April 2006 *Shaukat Khan*

Contents

1

Introduction

In the 1990s, progress in particle accelerators and storage rings resulted in ever higher beam intensities. Several particle "factories" came into existence – the B-meson factories at SLAC (Stanford, USA) and KEK (Tsukuba, Japan) as well as the Φ factory DAΦNE[1] (Frascati, Italy). Their design goal was the production of mesons and leptons at a high rate. Efforts to increase the "luminosity", defined as the particle production rate divided by the cross section, were also made at existing facilities. Furthermore, several synchrotron radiation sources of the "third generation" were designed and built in order to meet the ever-increasing demand on intense radiation in the vacuum-ultraviolet (VUV) and X-ray regime.

Traditionally, the design of a particle accelerator or storage ring starts with the layout of the external electromagnetic fields that act on each individual particle in the same way. These include the static magnetic fields to guide and focus the beam as well as radio-frequency (rf) fields for acceleration. However, in view of the present tendencies towards higher beam current and smaller beam cross section, effects caused by the coaction of many particles become more and more important. These "collective effects" can easily prevent the prospective intensity gain of a new facility if efficient countermeasures are not considered at an early stage of the machine design.

This book is meant as an introduction to collective phenomena and how to avoid or counteract them.

1.1 Definition of the Topic

Collective phenomena in particle accelerators and storage rings are those phenomena that are caused by the interaction between all beam particles, or at least a large number of them. Apart from beam particles, other particles may

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[1] Addresses of all facilities and institutes mentioned in this book are given in Appendix H.

exist inside the vacuum vessel of a machine and contribute in some way to collective effects: atoms and molecules of the residual gas, photons, photo electrons, or dust particles. However, the origin of the phenomena considered here is the joint action of the beam particles themselves.

Synchrotron radiation sources are – presently – storage rings for ultrarelativistic electrons or positrons emitting spontaneous electromagnetic radiation when the beam is transversely deflected by magnetic fields. These machines are optimized for ultraviolet, VUV and X-ray radiation, but their potential in other regimes such as THz radiation has been demonstrated as well. Several "third-generation" synchrotron light sources were built during the 1990s, others are under construction or planned. As an instructive example, BESSY II (Berlin, Germany) is described in Appendix B. Future synchrotron radiation sources are briefly discussed in Chap. 5. Restricting the discussion of collective phenomena to this class of electron[2] storage rings has several implications:

- The beam energy is constant. Effects occurring during acceleration in linear accelerators or circular machines are not considered.
- In a synchrotron radiation source, only one particle beam is present. Beam–beam interaction, an important topic for colliding-beam machines as used in elementary particle research, is not covered by this book.
- The particle speed is very close (and for practical purposes considered equal) to the speed of light.
- Electrons lose energy due to the emission of radiation. This energy is recovered when traversing a radio-frequency (rf) field at a particular phase. The phase restriction causes the electron beam in a storage ring to be a sequence of bunches much shorter than their spacing. The properties of a continuous ("coasting") beam will not be discussed.

The definition of the topic is not as restrictive as it may appear at first glance. Due to the large number of existing and planned facilities and their wide range of applications, synchrotron radiation sources play a dominant role in accelerator physics. Furthermore, they are very well suited for the discussion of collective phenomena, which strongly influence the quality of synchrotron radiation delivered to the users. Much of what will be said applies equally to other storage rings. There is, in particular, a similarity between synchrotron radiation sources and modern meson factories as far as beam energy (1–8 GeV), electron density (typically 1 nC per electron bunch) and bunch spacing ($\geq$2 ns) are concerned.

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[2] If no explicit distinction is made between electrons and positrons, everything said about "electrons" applies to both.

1.2 Outline

The collective phenomena occurring in synchrotron radiation sources include:

1. *Beam instabilities*. Beam particles interacting with their surroundings create electromagnetic fields that act on subsequent beam particles and can cause them to oscillate about their equilibrium position. One can distinguish between
   - short-range fields acting within an electron bunch and long-range fields acting between different bunches.
   - longitudinal oscillations, i.e. motion along the beam axis accompanied by periodic energy changes, and transverse oscillations, i.e. motion perpendicular to the beam axis.
2. *Lifetime-limiting effects*. The lifetime of an electron beam in a storage ring is governed by effects that may be classified as “collective” due to their dependence on the beam current
   - Elastic (Coulomb-) scattering of electrons on residual gas nuclei or shell electrons causes an angular deflection. On the other hand, the emission of bremsstrahlung photons leads to a loss of energy. Both may cause the interacting beam electron to be lost. The residual gas density depends strongly on the number of synchrotron radiation photons hitting the walls of the vacuum vessel, which in turn depends on the beam current.
   - Møller scattering between two electrons within the same bunch (intrabeam scattering) causes an exchange of energy that may lead to the loss of both electrons. This is the “Touschek effect”, which is the most severe lifetime-limiting factor in third-generation synchrotron radiation sources.
3. *Ion effects.* If the residual gas density is high, e.g. during the commissioning phase of a new storage ring or after the vacuum vessel was vented, positive ions are created and trapped by stored electrons (not by positrons!) and may lead to beam instabilities. Because of their minor importance under normal operation conditions, ion effects will be discussed in the appendix.

These collective phenomena will be discussed in some detail, often using the synchrotron radiation source BESSY II as an example. Chapter 2 is dedicated to beam instabilities and Chap. 3 to lifetime issues. In Chap. 4, countermeasures employed at BESSY II and other facilities will be described. Finally, in Chap. 5, some conclusions for present-day and future synchrotron radiation sources will be drawn.

This text is complemented by an extensive appendix where more detailed information can be found. Throughout the book, SI units [1] will be used. Without further mention, c is always the speed of light ($2.998 \times 10^{8}\,\mathrm{m\,s^{-1}}$), e is the elementary charge ($1.602 \times 10^{-19}\,\mathrm{C}$), ε_0 is the permittivity of free space ($8.859 \times 10^{-12}\,\mathrm{F\,m^{-1}}$), μ_0 is the permeability of free space ($1.257 \times 10^{-6}\,\mathrm{N\,A^{-2}}$), and i symbolizes $\sqrt{-1}$.

2

Collective Instabilities

2.1 Overview

Charged beam particles in a storage ring interact with the surrounding walls of the vacuum chamber and create electromagnetic fields that can act back on the particles themselves. These fields disturb their intended longitudinal and transverse position given by the magnetic guide fields and the electromagnetic radio-frequency (rf) fields. If, in turn, the disturbed particles create fields that tend to enhance the disturbed motion, a "collective instability" occurs. This must not be confused with disturbed particle trajectories due to optical resonances or nonlinear components of the guide field, which are not of a collective nature.

Collective instabilities show up as longitudinal or transverse beam excitations, where individual bunches or parts of bunches oscillate against each other. These oscillations may remain limited in amplitude, they may rise and collapse irregularly or periodically (sometimes in a sawtooth fashion), or they may increase in amplitude until the beam is lost.

Beam instabilities impair the quality of synchrotron radiation. One important figure of merit is the brilliance or spectral brightness[1] B:

$$B = \frac{\dot{N}_\gamma}{4\pi^2\, \sigma_x\, \sigma_y\, \sigma_{x'}\, \sigma_{y'}\, \mathrm{d}E/E_\gamma} \quad \left(\frac{\text{photons s}^{-1}}{\text{mm}^2\ \text{mrad}^2\ 0.1\%\,\text{bandwidth}}\right), \tag{2.1}$$

where σ_x, σ_y, $\sigma_{x'}$ and $\sigma_{y'}$ are standard deviations of the horizontal and vertical spatial and angular distributions, respectively. By convention, the photon flux $\dot{N}_\gamma$ is integrated over a photon energy range ("bandwidth") of $\mathrm{d}E/E_\gamma = 0.001$ or 0.1%. Instabilities limit the brilliance in different ways:

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[1] The term "brilliance" tends to be more popular in Europe. Depending on the author, "brightness" may denote the photon flux per bandwidth and solid angle, or the photon flux per area and solid angle; whereas "spectral brightness" is synonymous with brilliance. To increase the confusion, these quantities may or may not be normalized to the electron beam current.

- A transverse oscillation of electron bunches increases the time-averaged size and angular divergence of the radiation source.
- An oscillation in longitudinal phase space implies an oscillation of the electron energy E, which increases the time-averaged spread of photon energies in the undulator line spectrum, since the photon energy of undulator radiation is proportional to E^2.
- Instabilities can be a serious limitation of the storable beam current I and consequently of the photon flux $\dot{N}_\gamma \sim I$.

It is instructive and for many applications sufficient to describe the longitudinal or transverse motion as a harmonic oscillation

$$A(t) = A_\circ \exp(-\mathrm{i}\,[\Omega + \Delta\Omega]\,t)\ . \tag{2.2}$$

The frequency shift $\Delta\Omega$ is a complex number. Its real part describes a modification of the oscillation frequency, whereas its imaginary part denotes the rate of change of the oscillation amplitude. The signature of an instability is an increasing amplitude, the imaginary part of $\Delta\Omega$ being positive.

For electrons travelling nearly at the speed of light, the electromagnetic fields created by the interaction with the surrounding walls act almost exclusively on trailing electrons, and they are figuratively called *wake* fields. In a storage ring, however, an electron can be influenced by the wake field of trailing particles from previous revolutions.

Instead of fully characterizing the wake field created by a given charge at each point in space and time, it is usually sufficient to specify the voltage experienced by a trailing “test” charge as function of its distance to the leading charge, integrated over a certain period of time (e.g. the time it takes to traverse significant structures like rf cavities). The Fourier transform of this (real) wake function is a complex function of frequency. This function is called *impedance*. In principle, impedance and wake function are equivalent descriptions of the same reality, one given in the frequency domain, the other in the time domain. Their usage, however, depends on the respective application. The wake function, for example, is better suited for time-dependent simulations, whereas analytical calculations are often simpler in the frequency domain since the beam motion is periodic and certain parameters (like the skin depth in the wall of a vacuum chamber) depend on frequency.

Impedance, wake function and the beam instabilities caused by them can be classified according to the following criteria:

- *Longitudinal* wake fields influence the energy of beam particles and their spatial distribution along the beam axis. *Transverse* wake fields deflect beam particles perpendicular to the beam axis in horizontal or vertical direction. Consequently, there are longitudinal and transverse wake functions, impedances and instabilities.

- *Long-range* wake fields decay slowly enough to act on trailing particle bunches. They correspond to *narrow-band* impedances in the frequency domain. They may decay over many revolution times of the storage ring causing *multi-bunch* instabilities where all bunches act like an ensemble of coupled pendula.

  Short-range wake fields correspond to *broad-band* impedances and are, by definition, only significant within a particle bunch. Oscillations within a bunch are called *single-bunch* instabilities.
- There are distinct differences between beam instabilities in *linear accelerators* and *storage rings* as well as between oscillations of a *continuous beam* (also called *coasting beam*) and a *bunched beam*.

Concentrating on synchrotron radiation sources, this book is restricted to instabilities of bunched beams in storage rings. Under ordinary circumstances, only two types of instabilities are significant:

- *"Robinson"-type instabilities* are predominantly multi-bunch oscillations. The forces resulting from long-range wake fields create an imaginary frequency shift that is proportional to the beam current. This kind of instability exists at all values of beam current without any threshold. In practice, however, a threshold is given by the current above which the rise time of the instability is shorter than the damping time of stabilizing mechanisms (see Sect. 2.5). This instability type can essentially be understood using a one-particle model where each bunch is represented by a "macro particles".
- *Mode-coupling instabilities* are single-bunch phenomena. Their model description requires to split a bunch into two or more macro particles, or to treat it as a continuous charge distribution. At low bunch current, the wake fields cause a current-dependent shift of the real frequency of each mode of oscillation. A non-zero imaginary frequency shift, that describes an instability, occurs only above a certain threshold current at which the real frequencies of two modes merge.

An example of a phenomenon that does not quite fit into these two categories is the "fast" ion instability, which is addressed in Appendix G. So far, it was only unambiguously observed in experiments where the residual gas pressure was intentionally increased.

Viewed in the frequency domain, instabilities can occur when the impedance is non-zero at a frequency that corresponds to a possible oscillation mode of the electron beam. Therefore, the frequency spectrum of a beam is described next. What follows is an introduction to wake functions and impedances, and how to predict them. Subsequently, different instability models with increasing degree of complexity are discussed, followed by a description of "natural" damping mechanisms. Finally, methods to measure the impedance at storage rings are addressed. The active control of collective instabilities using feedback systems is the topic of a later chapter.

2.2 The Frequency Spectrum of a Stored Beam

The beam spectrum, i.e. the beam current as function of frequency is deduced from the current as function of time, detected at a fixed position of the storage ring. This is analogous to the impedance, which is usually linked to geometrical or electrical properties at a fixed position of the vacuum vessel. Oscillations of the electron bunches create additional lines in the beam spectrum. Their frequencies depend not only on the oscillation frequency but also on the phase differences between subsequent bunches. Excitation of a particular oscillation requires an impedance component at its frequency to be present.

The beam spectrum $J(\omega)$ and the beam current as function of time $j(t)$ are connected via Fourier transform:

$$\begin{aligned} J(\omega) &= \int_{-\infty}^{\infty} \mathrm{d}t \, j(t) \exp(-\mathrm{i}\omega t), \\ j(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{d}\omega \, J(\omega) \exp(\mathrm{i}\omega t) \; . \end{aligned} \tag{2.3}$$

The notion of a negative frequency[2] should not create any conceptual difficulty considering that only the real part of the oscillation is measurable:

$$\mathrm{Re} \exp(-\mathrm{i}\omega t) = \mathrm{Re} \exp(\mathrm{i}\omega t) = \cos(\omega t) \; . \tag{2.4}$$

Thus, a measuring apparatus (e.g. a spectrum analyzer) does not distinguish between negative and positive frequencies. The negative part of a spectrum may be thought of as being “flipped over” to the positive side of the frequency axis.

A measured spectrum also depends on the frequency characteristics of the detector and electronic circuitry. In the following description of idealized spectra, technical issues of this kind are ignored. A more detailed discussion of beam spectra is given e.g. in [2], instruments like spectrum analyzers and network analyzers are explained in [3].

2.2.1 Pointlike Electron Bunches

As a starting point, consider a beam of pointlike electron bunches without any longitudinal or transverse oscillation (see Fig. 2.1).

Case 1a. Let a pointlike electron bunch make a single passage past a (likewise pointlike) detector. The current signal (in arbitrary units) can be described using Dirac's δ-function

$$j(t) = \delta(t) \; . \tag{2.5}$$

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[2] Throughout this book, f denotes the frequency in terms of cycles per time unit, whereas $\omega = 2\pi f$ is the angular frequency, i.e. phase angle per time unit. For simplicity, the word “frequency” is used for both.

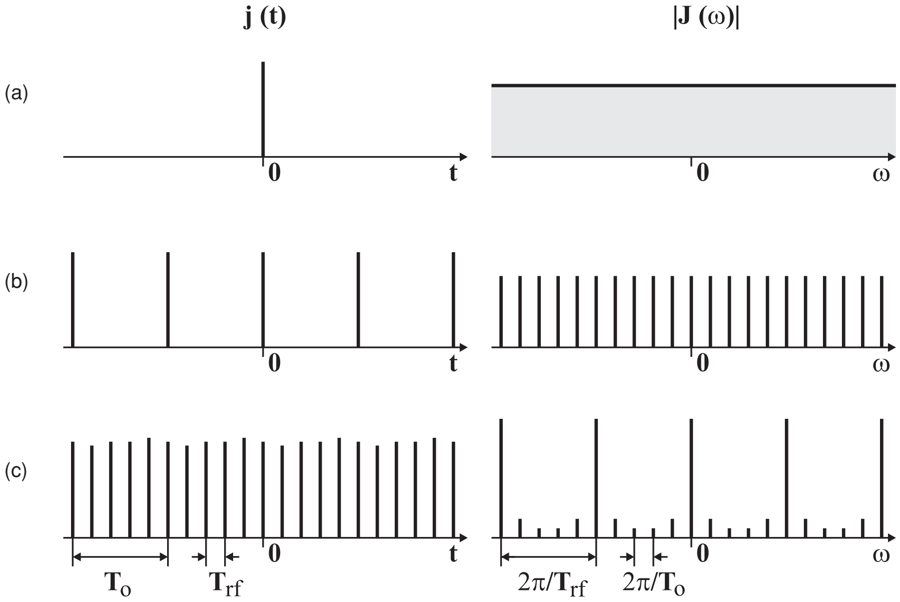

Fig. 2.1. Detected current signal caused by pointlike electron bunches as function of time (*left*) and frequency (*right*). (**a**) Single passage and (**b**) multiple passages of a single bunch with revolution time $T_\circ$, (**c**) multiple passages of $T_\circ/T_{\mathrm{rf}}$ equidistant bunches. Slight differences in bunch charge cause spurious revolution harmonics between the dominant rf harmonics

Fourier transformation results in an infinitely broad spectrum:

$$J(\omega) = \int_{-\infty}^{\infty} \mathrm{d}t\, \delta(t) \exp(-\mathrm{i}\omega t) = 1\ . \tag{2.6}$$

Case 1b. Let a pointlike electron bunch circulate in a storage ring with revolution time $T_\circ$ and revolution frequency $\omega_\circ = 2\pi/T_\circ$. The beam current as function of time corresponds to a sequence of equidistant δ-functions:

$$j(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_\circ)\ . \tag{2.7}$$

Its Fourier transform is

$$J(\omega) = \sum_{n=-\infty}^{\infty} \exp(-\mathrm{i}\omega n T_\circ) = \omega_\circ \sum_{p=-\infty}^{\infty} \delta(\omega - p\omega_\circ)\ , \tag{2.8}$$

where Poisson's sum rule for Fourier pairs $[f(t),\ F(\omega)]$ (e.g. [4])

$$\sum_{n=-\infty}^{\infty} f(a\,n) = \frac{1}{a} \sum_{p=-\infty}^{\infty} F\left(\frac{2\pi p}{a}\right) \tag{2.9}$$

was applied with $f = \exp(-\mathrm{i}\omega n T_\circ)$ and $a = T_\circ$. The electron bunch in the storage ring creates a spectrum of lines at integer multiples of the revolution frequency. These lines may be called "revolution harmonics". In order to understand this result qualitatively, consider (2.7) to represent periodic samples of a sinewave. These samples are consistent with an oscillation of frequency $\omega_\circ$, but any integer multiple frequency would yield the same sample values. This frequency ambiguity is called "aliasing".

Case 1c. With $\omega_{\mathrm{rf}} = h\,\omega_\circ$ being the rf frequency, h electron bunches correspond to a complete fill of the storage ring in the sense that each rf potential well (also called "bucket") is occupied, and h is called the harmonic number of the storage ring. The current given by pointlike bunches with a temporal spacing of $T_{\mathrm{rf}} = T_\circ/h$ is

$$j(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_{\mathrm{rf}}) \; . \tag{2.10}$$

In analogy to (2.8), the beam spectrum is given by

$$J(\omega) = h\,\omega_\circ \sum_{p=-\infty}^{\infty} \delta(\omega - p\,\omega_{\mathrm{rf}}) \; , \tag{2.11}$$

In this case, the lines of the spectrum are at integer multiples of the rf frequency and may be called "rf harmonics". In real storage rings, however, small inhomogeneities in the fill pattern create spurious lines at multiples of the revolution frequency $\omega_\circ$ as well.

2.2.2 Extended Electron Bunches

The unphysical result of a spectrum extending to infinite frequency is a consequence of the assumption of pointlike bunches. As a more realistic approach, let the longitudinal charge density of each electron bunch be a Gaussian with standard deviation σ_τ in units of time (see Fig. 2.2).

Case 2a. A bunch with Gaussian charge distribution that passes the detector only once creates a current signal (in arbitrary units) of

$$j(t) = \frac{1}{\sqrt{2\pi}\,\sigma_\tau} \exp(-t^2/2\,\sigma_\tau^2) \tag{2.12}$$

and its spectrum is given by

$$J(\omega) = \exp(-\omega^2\sigma_\tau^2/2) \; . \tag{2.13}$$

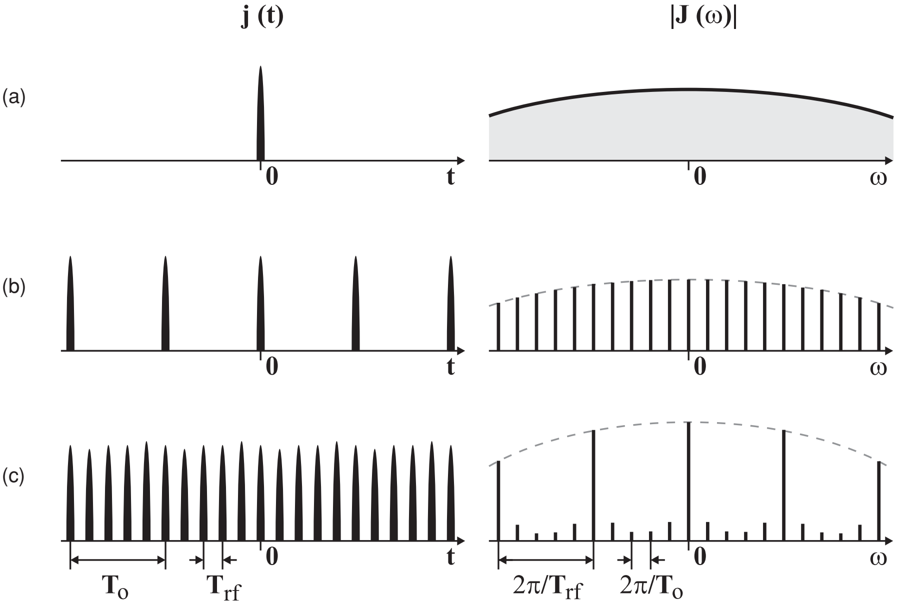

Fig. 2.2. Detected current signal caused by longitudinally extended bunches as function of time (*left*) and frequency (*right*). (**a**) Single passage and (**b**) multiple passages of a single bunch with revolution time $T_\circ$, (**c**) multiple passages of $T_\circ/T_{\mathrm{rf}}$ equidistant bunches

Case 2b. For a Gaussian bunch in a storage ring with revolution time $T_\circ$ and revolution frequency $\omega_\circ = 2\pi/T_\circ$, the beam current is a convolution of the previously assumed sequence of δ-functions and the charge density distribution:

$$\begin{aligned} j(t) &= \frac{1}{\sqrt{2\pi}\,\sigma_\tau} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \mathrm{d}t'\, \delta(t' - nT_\circ)\, \exp(-[t-t']^2/2\,\sigma_\tau^2) \\ &= \frac{1}{\sqrt{2\pi}\,\sigma_\tau} \sum_{n=-\infty}^{\infty} \exp(-[t - nT_\circ]^2/2\,\sigma_\tau^2)\ . \end{aligned} \tag{2.14}$$

According to the convolution theorem, its Fourier transform is simply given by the product of the Fourier transforms of the convoluted functions:

$$J(\omega) = \omega_\circ \exp(-\omega^2\sigma_\tau^2/2) \sum_{p=-\infty}^{\infty} \delta(\omega - p\omega_\circ)\ . \tag{2.15}$$

The finite extension of the electron bunches defines the envelope of the line spectrum: the shorter the bunches, the broader the spectrum. The line width in the spectrum, on the other hand, is determined by the envelope of the distribution in time: the longer the measurement time (here assumed to be infinite), the smaller the line width.