Crimes and Mathdemeanors

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Preface

> The great book of nature lies ever open before our eyes and the true philosophy is written in it … But we cannot read it unless we have first learned the language and the characters in which it is written … It is written in mathematical language….
>
> —Galileo Galilei

Quotes like the one above convey the immense value of mathematics as an indispensable tool in understanding the laws that govern our physical world. However, they also make math sound serious and daunting and intimidating. I like math not because it is valuable, but because it is fun and because it is beautiful.

I love the "Aha" feeling as a flash of insight suddenly illuminates the solution to some problems. I also love the sense of exhausted victory as a relentless mental effort provides the solution to others. Mostly, though, I love how math is constantly full of surprise, sometimes completely defying my intuition and overturning my common sense.

When I was in grade school, my favorite books to read were the *Encyclopedia Brown* series by Donald J. Sobol. I was amazed at how Encyclopedia Brown, using a mixture of knowledge and logic, was able to solve mysteries that

vexed others. As I got older, and became involved in mathematics, I began to envision a kid detective like Encyclopedia who would solve mysteries based not just on logic, but also on serious mathematics. I started playing with a few story ideas to see if such a concept would actually work. The book that you have before you is my attempt to render such a character and to convey the sense of mystery and joy that I find in mathematics. Interestingly, during the process of writing, a popular television series, *Numb3rs*, debuted, giving me hope that the idea of a mathematical detective may indeed be appealing to a broad audience.

My intended audience is my friends and others like them—young people who like math, but probably not enough to just sit and read mathematics texts or work through problem books. Yet, they may be amenable to thinking mathematically if it is within the context of solving a mystery or a puzzle—if a challenge is laid before them to match wits with the protagonist of the stories, Ravi. Therefore, the mathematics used in this book is mostly of the high-school level, with the hope that no reader will feel that the mathematics is beyond his or her understanding.

My use of the name "Ravi" for the young mathematical detective who solves these mysteries probably also deserves some comment. He is named after Ravi Vakil, a faculty member of the mathematics department at Stanford University. As I got more "serious" about math, one of my favorite books was Dr. Vakil's *A Mathematical Mosaic: Patterns & Problem Solving* (Brendan Kelly Publishing, Burlington, Ontario, 1997). It has just the right flavor of fun and rigor and has very interesting profiles of young mathematicians that Ravi Vakil had come to know. Thus, I decided to name my hero after Dr. Vakil. I must clarify that

Dr. Vakil does not know me, and I never consulted with him before naming my detective. I sincerely hope that he does not mind my choice.

The problems around which the detective stories in this book are built come from many sources, which I have tried to reference in a separate index. I highly recommend all of these books to anyone with some interest in mathematics. Unfortunately, over a few years of reading about mathematics and solving math problems, I sometimes could remember a problem but not where I had seen it. Whenever I could find the source of the problem at the heart of a story, I have cited it. Where I could not, I ask the forgiveness of those authors whom I did not credit. Another complicating factor is that problems are often found in many books, especially when they have become popular and part of the "lore" of mathematics. This makes it difficult to assign original credit to any one person for a problem. In any case, I make no pretense of developing most of these problems myself—only a few are my own. I have, however, adapted them as needed and placed them within the context of interesting and novel stories. Also, I have tried to present the solutions in an original fashion and one suited to my intended audience.

I hope you, the reader, find in this book some of the pleasure that I had in writing it. If it does for some what *A Mathematical Mosaic* did for me—igniting a spark of enthusiasm about mathematics—I will be both humbled and proud.

Acknowledgments

I realize that it is unusual for someone my age to write a book, let alone a mathematics book. It would not have been possible without the encouragement of my parents, as well as the tremendous support of my wonderful publisher, Klaus Peters. I am very grateful to Dr. Katherine Socha for reviewing the entire manuscript in detail and presenting many helpful suggestions as well as pointing out and correcting errors as they occurred. Also, I sincerely thank Dr. Karl Hofmann for also reviewing the stories and providing many helpful comments, and most especially for his wonderful and vibrant illustrations that truly brought the stories to life. To Ms. Charlotte Henderson, I owe an inestimable debt of gratitude. She shepherded the manuscript to production, with meticulous attention to detail. She reviewed, edited, and clarified where needed, and she also redrew all of the figures. The book is prettier, sharper, crisper, and better because of her extensive efforts.

Finally, I would like to especially thank my father. When he read my first stories, he was the one who suggested that the idea of a mathematical Encyclopedia Brown-like detective had the makings of a book. He is a physician who, at the time, was engrossed in writing a medical textbook. He encouraged me to pursue the idea of a book, promising that we would find the discipline to work together, I on my book and he on his. He kept his promise.
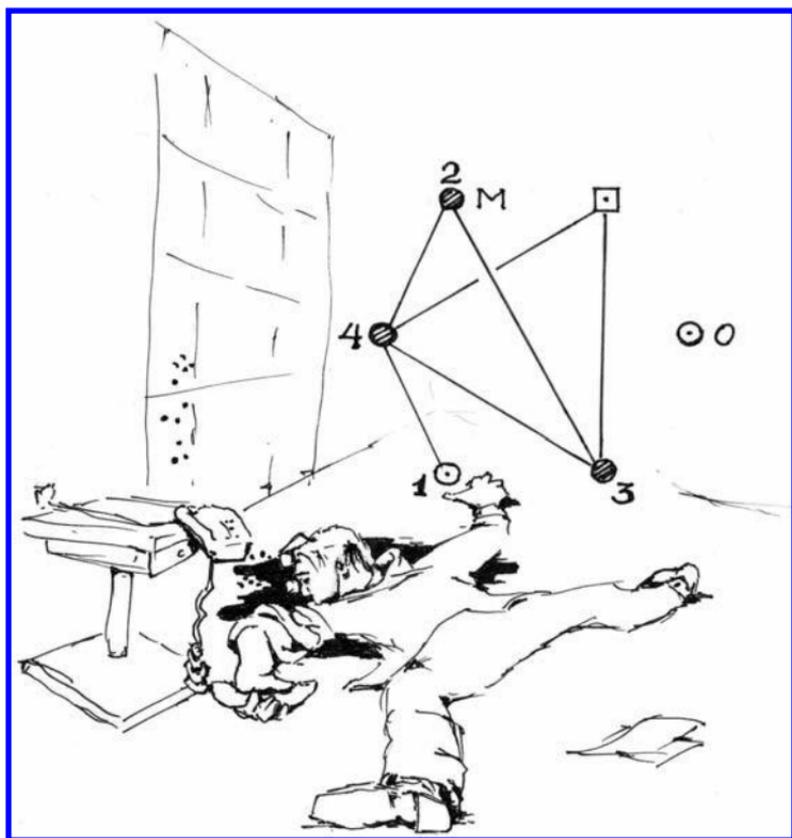
A Mystery on Sycamore Lane

Ravi strode onto the large, neatly manicured lawn of 423 Sycamore Lane. Two police cars with flashing lights stood in the driveway of the large white house with tall columns at its entrance. The policemen nodded to Ravi as he walked by. He was a slender fourteen-year-old boy with curly brown locks and large, inquisitive brown eyes. While it was not common for the police to let a teenager stroll into the scene of a homicide investigation, Ravi was no common teenager. He was, by all accounts, a genius, though he was far too modest to think of himself in that way. Rather, he thought of himself as someone who loved puzzles, and loved thinking through problems, especially if they had a mathematical flavor. His uncanny ability to do just that had earned him the trust of Chief Dobson, head of the Criminal Investigations Unit of the Chicago Police Department. Chief Dobson had come to know Ravi through Andy, his son and Ravi's classmate in tenth grade. While Ravi was at the Dobsons' house for dinner one evening, the Chief was recounting the details of an especially difficult case. Upon hearing the case, Ravi was able to solve it with just a few minutes of thinking. Since then, Chief Dobson made it a habit to consult Ravi when faced with a case that was especially puzzling.

Ravi quickly bounded up the four steps that led to the half-open large carved wooden door of the residence and walked into the marble entry.

"Hello Ravi. Good to see you," said Chief Dobson.

"Hello Chief," replied Ravi. "What have we got?"

Chief Dobson motioned Ravi to follow him into the living room.

"This is Dr. and Mrs. Arden. Unfortunately, a murder was committed in their house today," said Chief Dobson.

"I'm sorry to hear that," responded Ravi. "How can I help, Chief?"

"Well, the victim was a man named Dr. Rosmoyne, a friend of the Ardens. He was killed in the study with a single shot to the back of the head early this afternoon. Apparently, he was on the phone with his back to the door, since the receiver was dangling from its cord next to him when he was found."

"Who found him?" asked Ravi.

"I did," answered Dr. Arden. Ravi turned to face Dr. and Mrs. Arden, who were sitting clasping hands on a small sofa.

"Please tell me what happened," requested Ravi.

Dr. Arden began to recount the details of the barbecue lunch which had ended so tragically: "We invited some friends over for Sunday brunch. We invited Dan Rosmoyne, the Wentworths and the Finnegans. Our guests showed up between 11:00 and 11:30. We hung out between the backyard patio and the upstairs den. We just ate and talked and watched the basketball game, you know, just a typical lazy Sunday. I was out by the pool most of the time barbecuing, and Stacey (he looked at his wife) was back and forth between the kitchen and the den. Artie Wentworth and I were talking politics while I barbecued. The others were

upstairs in the den watching the game. I didn't see Dan for most of the party. He didn't like the sun much. Anyway, I thought he had left—some sort of emergency at the hospital, I thought."

"Did anyone see him leave?" asked Ravi.

"No, not really," answered Dr. Arden. Mrs. Arden also meekly shook her head no.

"Why did you think there was an emergency at the hospital, Dr. Arden?" asked Ravi.

"I think Bob Finnegan mentioned something when he came out to check how I was doing with the barbecue. He likes his steak extra well-done. I guess Dan asked him where our phone was because he needed to make a call, and Bob told him that the phone was downstairs in the study. Bob asked if that was okay, and I told him that it was. Then we all got caught up in the basketball game—the Bulls went into double overtime. When I noticed he wasn't there, I assumed he had run out to the hospital. That's the life of an obstetrician, you know. Babies just can't wait."

"So he was your colleague at the hospital, Dr. Arden?" inquired Ravi.

"No, no. I'm a professor at the university; I have a PhD in sociology."

"Then how did you know the victim, Dr. Arden?"

There was an awkward silence. Dr. and Mrs. Arden looked at each other. Dr. Arden took a deep breath and began, "He was—uh—our doctor. We were having trouble conceiving a baby, and we went to him. We've been sort of friends since, and we see each other from time to time. We were not very close or anything, but he was friends with our other guests, and his wife was out of town, so we invited him to join us."

"How did you find him, Dr. Arden?" asked Ravi, scratching his chin softly.

"A little while after the guests left, I walked into the study, and there he was lying face down on the ground in a pool of blood. It was horrible!"

Ravi continued his inquiry while Chief Dobson looked on, not wishing to disturb him, although many of these questions had already been asked and answered, "Was there anyone else in the house? A maid, perhaps, or children?"

"No," answered Dr. Arden, "Sunday is the maid's day off. And, as I mentioned, we don't have kids. The Finnegans left their kids at home, and the Wentworths don't have kids either."

"But Julie Wentworth is pregnant," interjected Mrs. Arden.

"Do you remember which of your guests left first, Mrs. Arden?" asked Ravi.

"They all left together," answered Dr. Arden.

"Are you sure?" Ravi inquired.

"Yes. Stacey and I walked them out to their cars in the driveway and we shook hands and they left. I'm usually a bit absent-minded, but this I remember quite distinctly, because just before they got into their cars, I happened to ask the Finnegans and the Wentworths, as well as Stacey, how many hands they had just shaken—it's for a project I'm working on about shifts in our social customs—and everyone gave me a different answer, which I thought was rather curious."

Ravi's eyebrows raised slightly and he said distractedly, looking off into the air, "Do you remember those answers?"

"No. That's where the absent-minded stuff comes in, you know."

"Yes, of course," said Ravi, turning to Stacey Arden, "Do you remember what they said, Mrs. Arden?"

"No, I don't," she answered.

"Do you remember how many hands you shook, then?"

"Yes. Four. I shook hands with all my guests," she answered after some hesitation.

"What else do we know, Chief?" asked Ravi, looking at Chief Dobson. "Have you talked with the Finnegans and the Wentworths?"

"Yes, of course. Some of my men are still at their houses. They gave pretty much the same story. Bob Finnegan says Dan Rosmoyne asked him about a phone, and Finnegan told him that the only phone he knew of was in the study, but there was also probably one in the Ardens' bedroom. Then, everyone got caught up in the game, and figured Dr. Rosmoyne had left."

"Did anyone hear a gunshot?" asked Ravi.

"No, apparently not. But they all say the TV was up loud and they were screaming at the players. Michael Jordan wasn't shooting too well but was hogging the ball," answered the Chief.

The Chief then cocked his head, indicating to Ravi that he wanted him to step out into the entry. The Chief followed him, and leaned in close, "We don't have a murder weapon. My men are over searching at the Finnegans and the Wentworths now. But we may have a motive. We found out that Bob Finnegan is an office administrator and that he used to work for Dr. Rosmoyne, but Rosmoyne had to let him go a couple of years ago when he switched his practice over to Mercy Hospital. Finnegan claims there were no hard feelings and that they were good friends. Mercy Hospital, apparently, has its own administrators, and that was the

reason Rosmoyne let him go. Also, Rosmoyne used to date Julie Wentworth, before meeting his wife. But she says that was a long time ago, and that they mutually decided to stop seeing each other, and that they've remained good friends since."

"Interesting," mused Ravi. "Do we have anything else?"

"Yes," continued the Chief, still leaning low and half-whispering. "We ran paraffin tests for gunpowder residue on everyone, figuring there would be gunpowder on the hands of the shooter. Bob Finnegan tested positive. But I think we've got some contamination or something. Julie Wentworth and Stacey Arden also tested positive, but everyone else was negative."

"Was Dr. Rosmoyne wearing a pager, Chief?" asked Ravi.

"What?"

"A pager, a beeper, like doctors wear. I know it's Sunday, but did he have his beeper?"

The Chief looked vexed, "I don't know. We didn't notice it, but I'll call downtown to the medical examiner's office and have them check right now. I've got a man over there."

The Chief pulled out his cell phone and began to dial. Meanwhile, Ravi walked out into the cool air of the early evening, and strolled pensively along the lawn, thinking and looking at the lush green grass beneath his feet.

The Chief came running out. "Good thinking, Ravi. He did have a small beeper in his pocket. They checked if he had pages, and he had gotten a page at 12:49. The pager was on vibrate, so I guess no one heard him get beeped. When my officer dialed the number on the pager, it was the Mercy Hospital operator. So, like I said, good thinking, but it doesn't really get us anywhere. We're still stuck."

"No we're not, Chief," said Ravi as he walked toward the edge of the driveway to retrieve his bicycle. Chief Dobson knew that Ravi had to be home for Sunday dinner. "I think the case is solved," said Ravi with a little smile as he straddled his bike.

"Solved?!" asked Chief Dobson, wide-eyed with disbelief.

"Yes," said Ravi. He began to pedal out of the driveway and onto the street. He turned his head and said to Chief Dobson, "The murderer is …"

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Now, it is time for you to match wits with Ravi. Do you know who killed Dr. Rosmoyne?

Analysis

Later that evening, after Ravi had finished dinner with his parents, Chief Dobson drove to Ravi's house and asked him how he had solved the case. Ravi smiled and said, "Dr. Arden actually solved it for me, Chief. The case hinged on a little problem in logic—on how many hands Stacey Arden must have shaken."

Ravi continued, "Let's say you and your wife have a dinner party, and invite two other couples over. After dinner, you both walk them to the door, and people shake hands goodbye. Of course, a husband doesn't shake his own wife's hand, and vice versa. You happen to ask each of your guests as well as your wife how many hands they shook. Now, suppose each person gives you a different answer."

"Okay," said the Chief.

"Then how many hands did your wife shake?" asked Ravi.

"What?" answered the Chief, as if he did not comprehend the question.

"How many hands did your wife shake, Chief?" repeated Ravi.

"I don't know. You didn't tell me that. How can I possibly figure that out? I don't have enough information. Any way, what does it matter? What does this have to do with our case?"

📍

Ravi was able to distill the details of the case into a single problem, the solution of which is also the solution to the case. Chief Dobson could not figure out the problem. If you can figure it out, you will solve the case as well!

Solution

To solve this problem, we need to "see through it." Here, it seems that there is not nearly enough information for Chief Dobson to determine how many hands his wife shook, or for us to determine how many hands Mrs. Arden shook. If we think carefully about the problem statement, though, a unique solution presents itself.

Since it seems that there is insufficient information, every piece of information we do have is critical. We are faced with a situation where Dr. Arden and his wife invite two other couples over. We know also that spouses don't shake hands. Now, two key facts need to be noticed:

1. The maximum number of hands which any person can shake is four. That is because there are six people total, made up of three couples. Person X does not shake hands with himself or his (or her) spouse, leaving a maximum of four possible handshakes.

2. Dr. Arden asked his wife and the two couples (five people total) about their handshakes and was quite definite that each person gave him a different answer. Since the maximum number of handshakes is four, and each of the five people shakes a different number of hands, the handshakes have to be distributed as zero, one, two, three, and four across Mrs. Arden and the two couples. We note that we have no information about how many hands Dr. Arden shook, since he did not ask himself any questions. Therefore, only the other five people need to be considered.

From these key facts, we can actually deduce another rule before moving forward. Consider the person who shakes
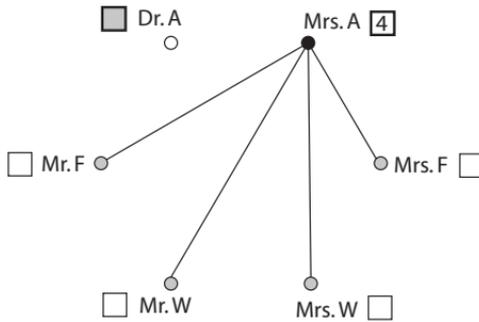
Figure 1. Mrs. A shakes four hands.

four hands. This person does not shake his own hand or the hand of his wife; therefore, he shakes hands with each of the four remaining people in the group. Then, this person's spouse must be the one who shakes zero hands, because everyone else has shaken the first person's hand. Therefore, as an additional principle, if someone has four handshakes then his (or her) spouse must have zero.

Now, we attempt to discover whether these facts provide us enough information to figure out how many hands Mrs. Arden actually shook. She claimed to have shaken four hands, and so we check that possibility first, using the diagram in Figure 1, where the individuals are identified by their initials.

If Mrs. Arden shook four hands, it is clear that none of the other guests could have shaken zero hands, since she shook hands with them all. Therefore, this possibility is eliminated.

Similarly, we can investigate whether Mrs. Arden could have shaken zero hands. In that case, one of the guests must have shaken four hands: let's say Mr. W. He cannot shake hands with Mrs. W, and so must have shaken hands with Mr. and Mrs. F and Dr. and Mrs. A. This contradicts the assumption that Mrs. A shook zero hands.